Project Management

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Definitions of a project

“A project is a series of activities directed to accomplishment of a desired objective”

“A project is a temporary and often customized initiative that consists of many smaller tasks and activities that must be coordinated and completed to finish the entire initiative on time and within budget”

“A project is a temporary endeavor involving a connected sequence of activities and a range of resources, which is designed to achieve a specific and unique outcome and which operates within time, cost and quality constraints and which is often used to introduce change”

Project management involves all activities associated with planning, scheduling, and controlling projects.

Plan your work first…..then work your plan!

Project

A (often unique) endeavor, which consists of a sequence of activities using the necessary resources to complete an objective in limited time, and with specific funds and defined quality standards.

Examples of projects

- All kinds of engineering projects (roads, buildings, bridges, factories, dams, etc.)
- Constructions of composite means of transport (cars, boats, airplanes)
- Conduct of events
- Pre-election campaigns
- Installation and maintenance of equipment
- Promotion of new products
- Development and implementation of information systems
- Conduct of surveys
- Organizations of sports competitions
- Entrance examinations
- Complicated surgeries

What is a project?

- Any unique endeavor with specific objectives
- With multiple activities
- With defined precedence relationships
- With a specific time limit for completion

Characteristics of a project

- A unique, one-time operational activity or effort
- Requires the completion of a large number of interrelated activities
- Established to achieve a specific objective
- Resources, such as time, capital, resources, and equipment, are limited
- Typically has its own management structure
- Needs leadership

Common features of projects

- The objective
- The life cycle (beginning, middle, end)
- The partial activities (which they are, priorities, relationships, who conducts them)
- The uniqueness
- The competition
- The available resources (human resources, equipment, funds, infrastructure)
- The time horizon (starting point and ending point)
- The project manager – The project execution team
- The client

Network analysis

Network analysis is the general name given to certain specific techniques which can be used for planning, managing and controlling projects.

Project scheduling

- Identifying precedence relationships
- Sequencing activities
- Determining activities' times & costs
- Estimating the requirements of resources (staff, material, equipment)
- Determining critical activities

Purposes of project scheduling

- Shows the relationship of each activity to the others and to the whole project.
- Identifies the precedence relationships among the activities.
- Encourages the setting of realistic time and cost estimates for each activity.
- Helps in better use of capital, human resources, and material resources by identifying the critical bottlenecks in the project.

What is project management?

- The application of a collection of tools and techniques to direct the use of diverse resources towards the accomplishment of a unique, complex, one time task with time, cost, and quality constraints.

- Its origins lie in World War II, when the military authorities of UK and USA used the techniques of operational research to plan the optimum use of resources.

- One of these techniques was the use of networks for the representation of a system with related activities.

Project management (PM) across the organization

- Accounting uses PM information to provide a timetable for major expenditures.
- Marketing use PM information in order to monitor the project progress and to provide updates to the customer.
- Information systems develop and maintain software which supports the monitoring of projects.
- Operations the information provided by PM in order to monitor the progress of the activities, both on and off the critical path, and to manage resources' requirements.

Design and scheduling of a project

Design incorporates the actions aiming at the determination of the activities, the teams which will carry them out, and the priority order in which the activities will be completed.

Scheduling is related with the development of a detailed project plan for the use of the resources in order to achieve the completion of the activities.
Incorporates activities such as:

- Determination of the starting and ending point of each activity
- Calculation of the duration of each activity
- Estimation of the total duration of the project
- Allocation of the necessary resources
- Readjustment of the resources
- Balance between project cost and project duration
- Comparison between the plan and the actual progress of the project
- Readjustment of the plan
- Review of the project

The six steps common to Project Management techniques

- Define the project and prepare the structure of the work breakdown.
- Develop the relationships among the activities.
  (decide which activities must precede and which must follow others)
- Draw the network connecting all the activities.
- Assign time and/or cost estimates to each activity.
- Compute the longest path through the network.
  This is called “critical path”.
- Use the network to help plan, schedule, monitor, and control the whole project.

Definition of terms in a network

Network

Is a combination of all project activities and events.

A powerful tool for designing and controlling a project.

- Graphical display of activities and events.
- Shows the dependency relationships among the activities (tasks) of a project.
- Clearly shows tasks that must precede (preceding) or follow (succeeding) other tasks in a logical manner.
- Clear representation of the project.

Activity

Is a minimum subdivision of the project, which requires times and resources for its implementation.

It is a time consuming effort required to perform a part of the project.

Event (also called node)

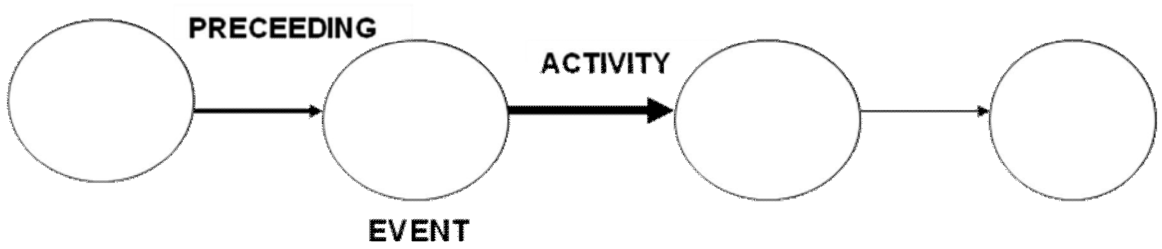
- Signals the beginning or end of an activity

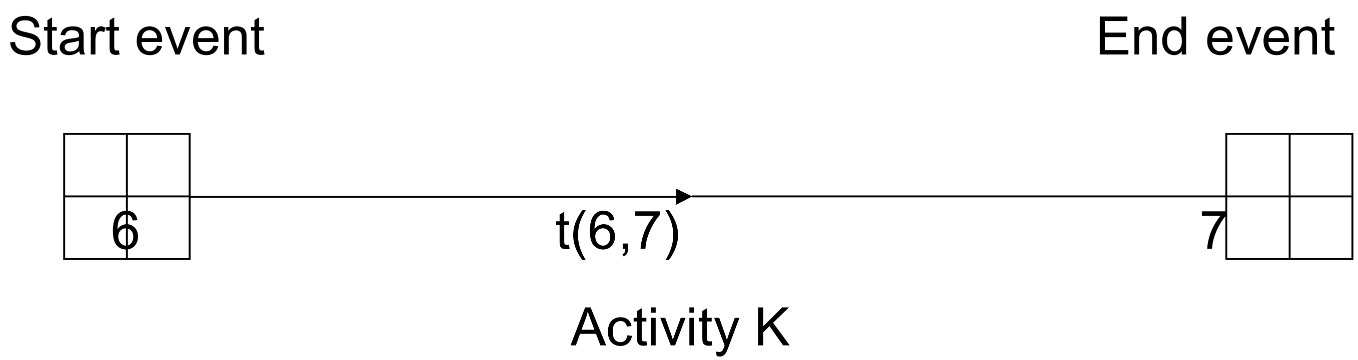
- Is an instantaneous point in time

- Designates a point in time

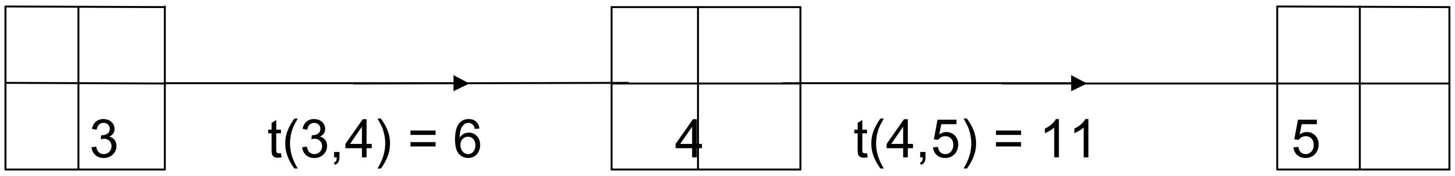
- Is represented by a circle (node)

Activities are represented by arrows and events by nodes.

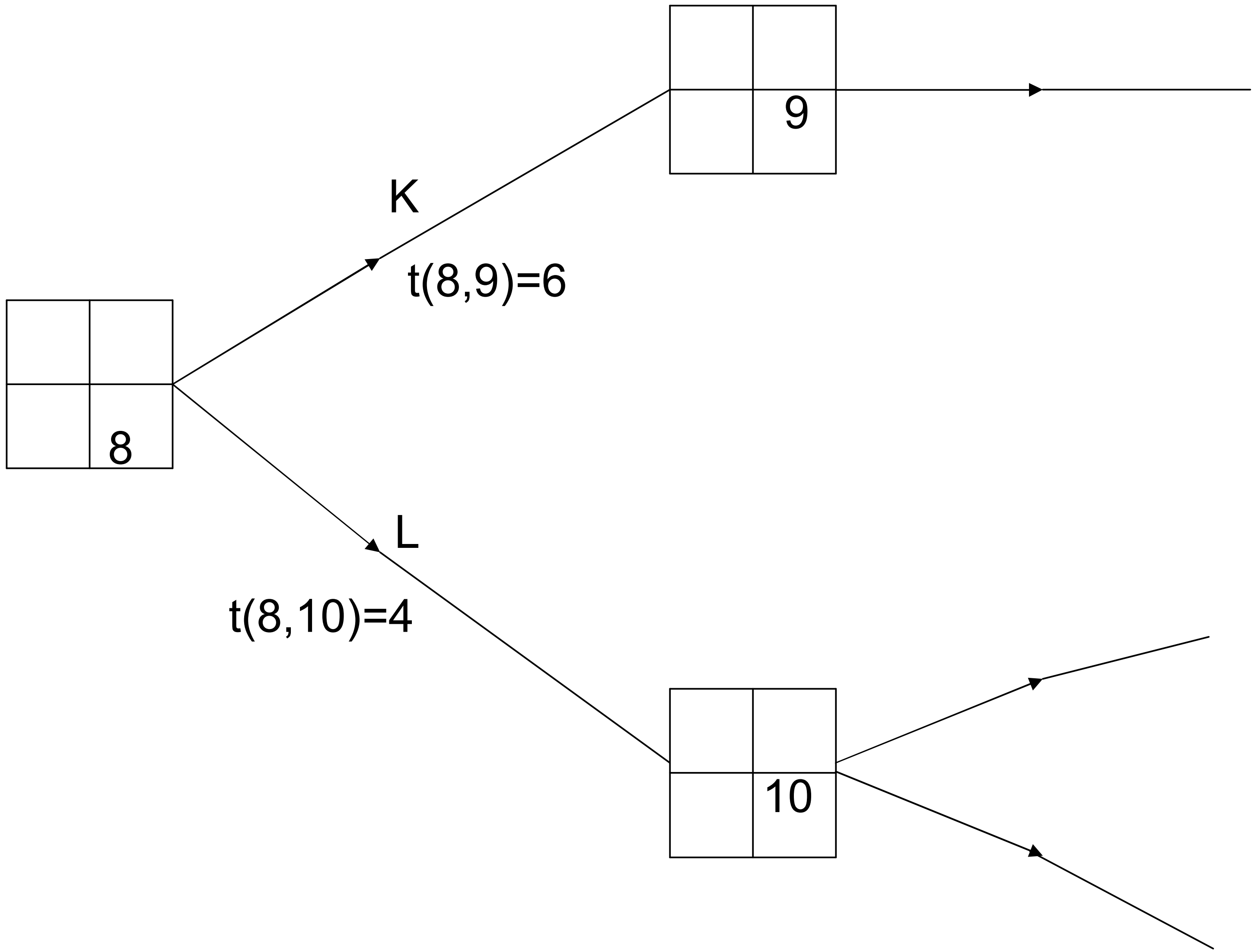




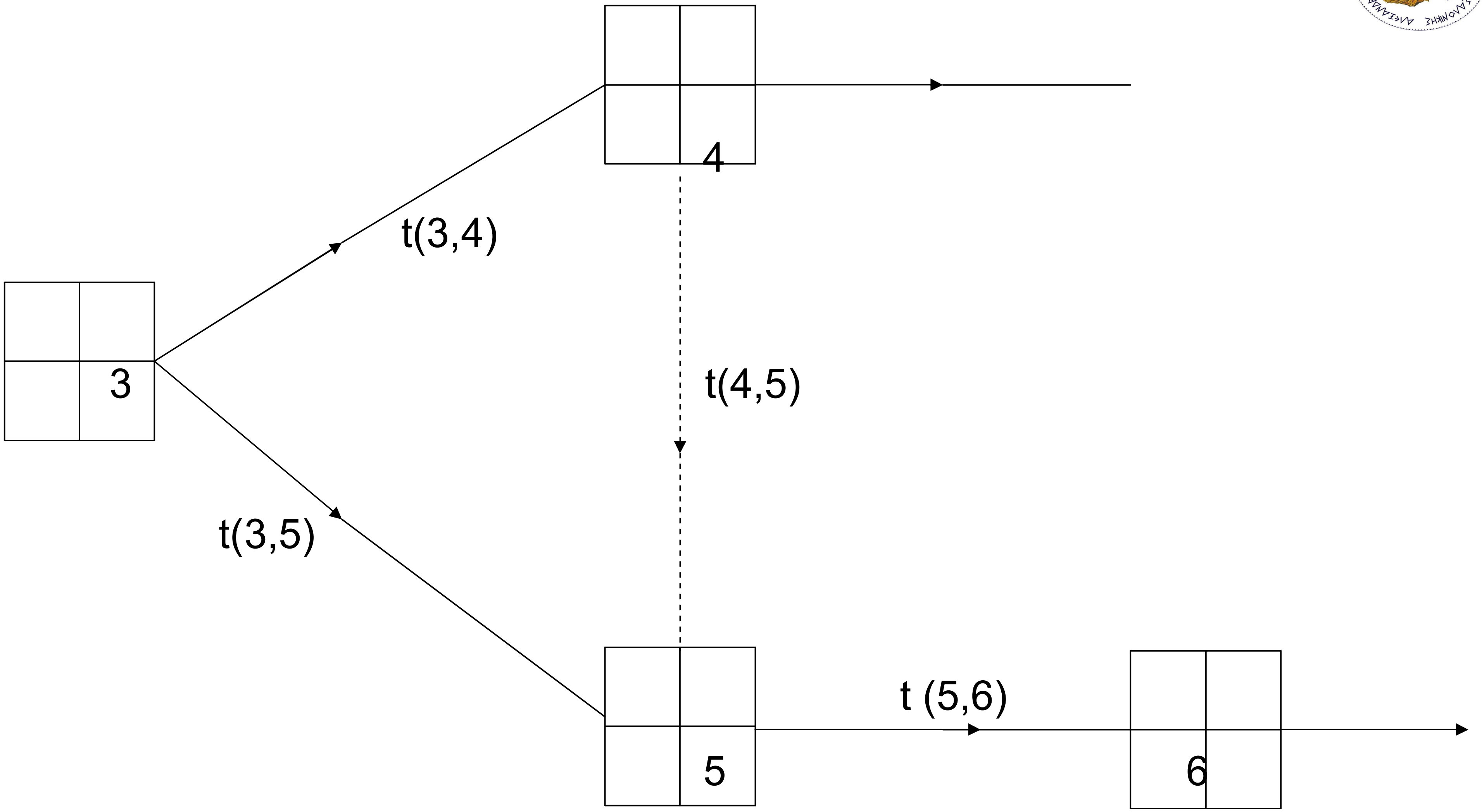
Graphical representation of an activity



Serial activities

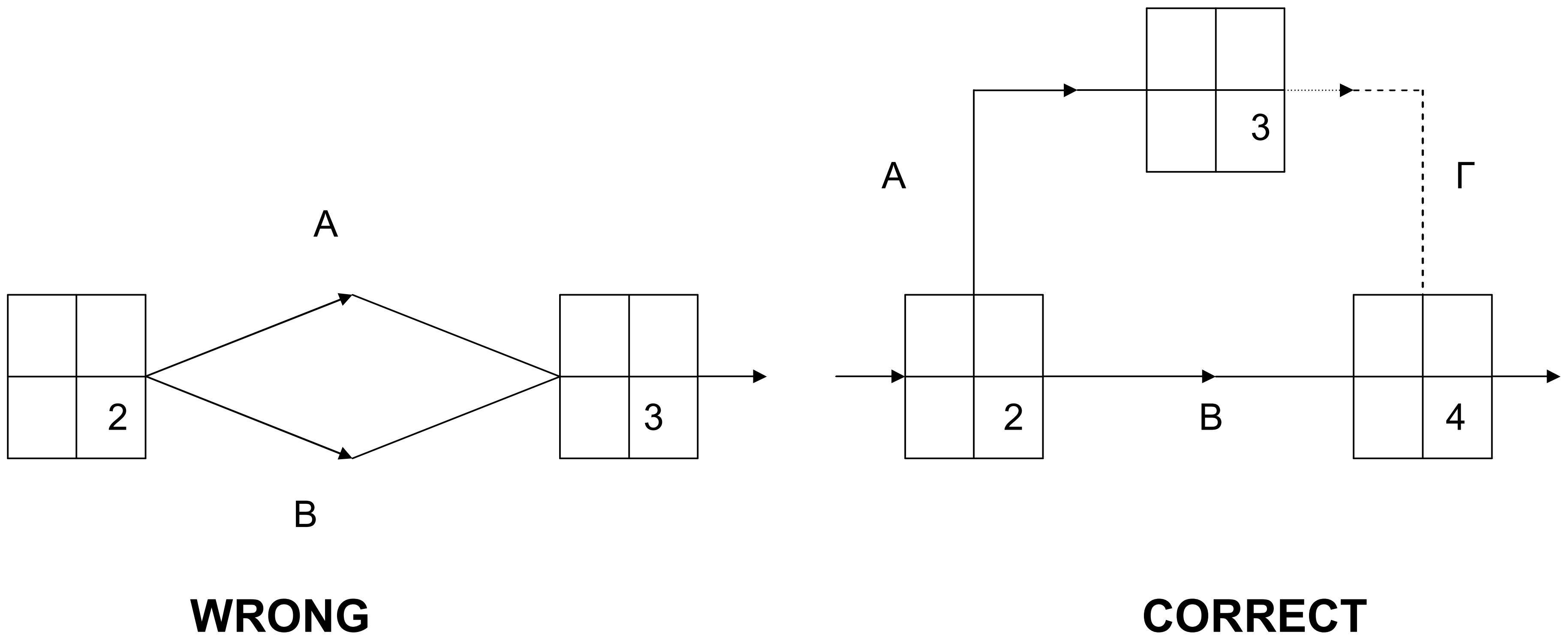


Parallel activities



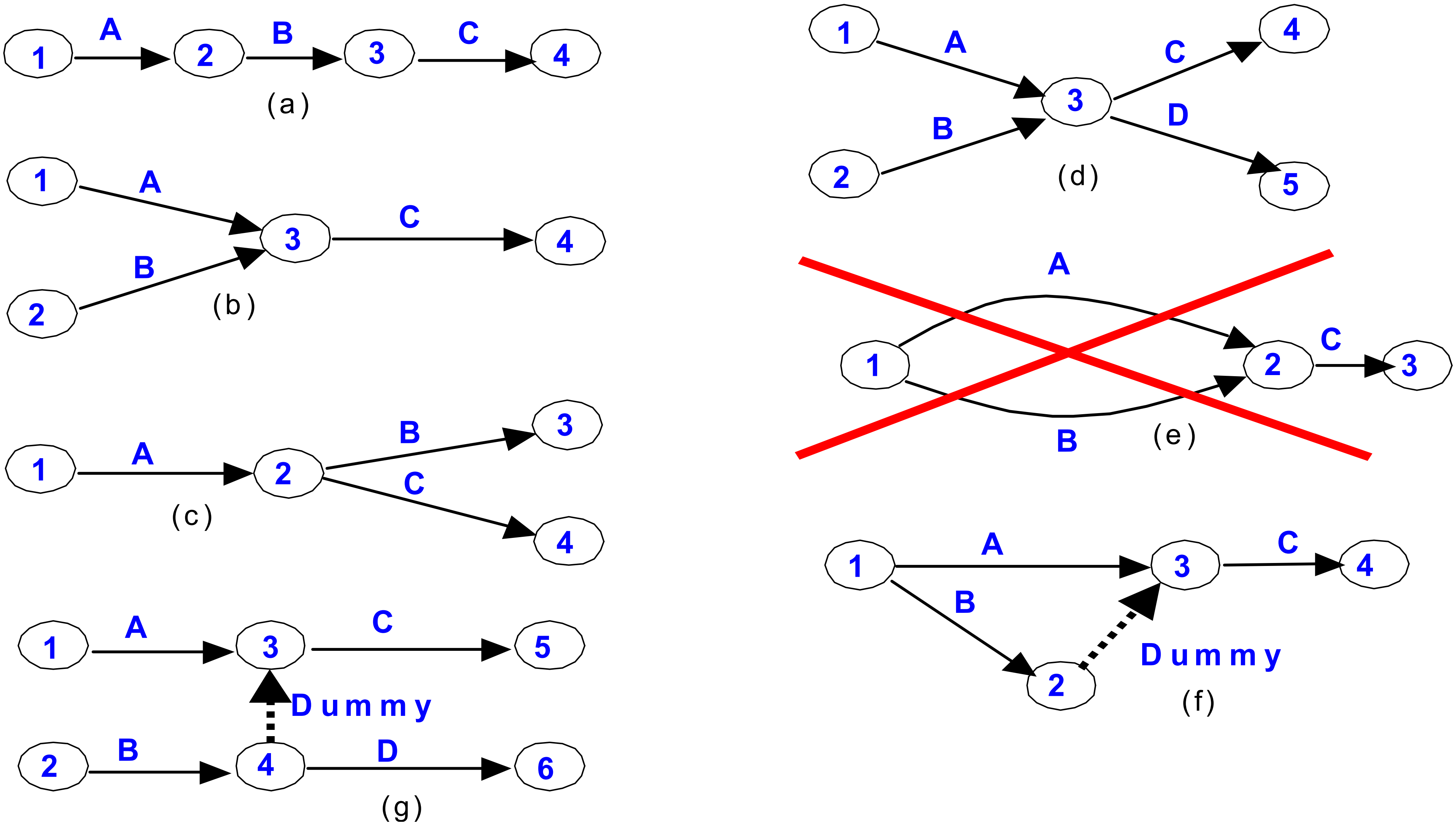
Fictitious activity

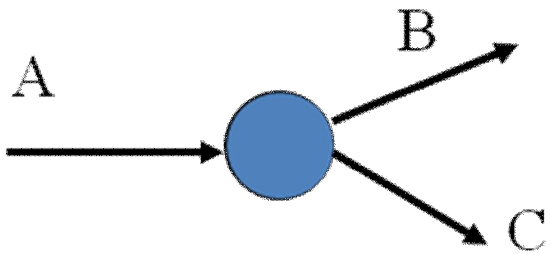
Activity (4,5) is fictitious and declares that (5,6) is serial with (3,4) as well



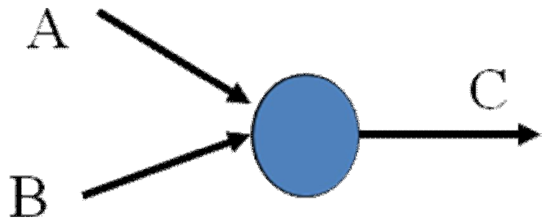
The addition of the fictitious activity (3,4) ensures that activities (2,3) and (2,4) are unequivocal

Activities’ situations in networks

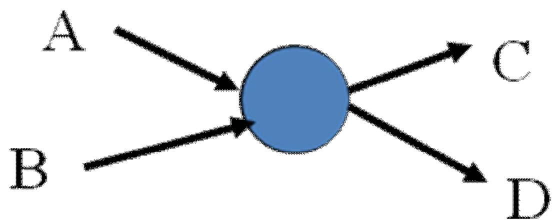




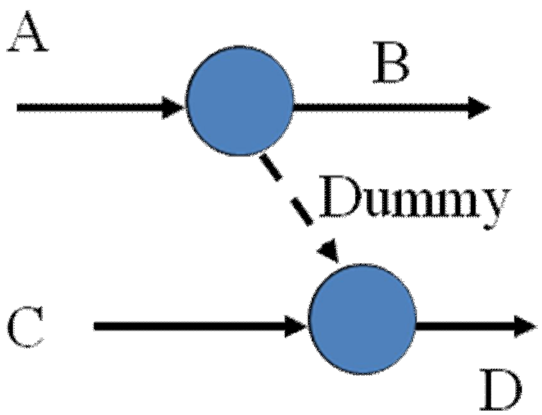
A must finish before either B or C can start



Both A and B must finish before C can start



Both A and C must finish before either of B or D can start



A must finish before B can start.
Both A and C must finish before D can start

Scheduling strategies

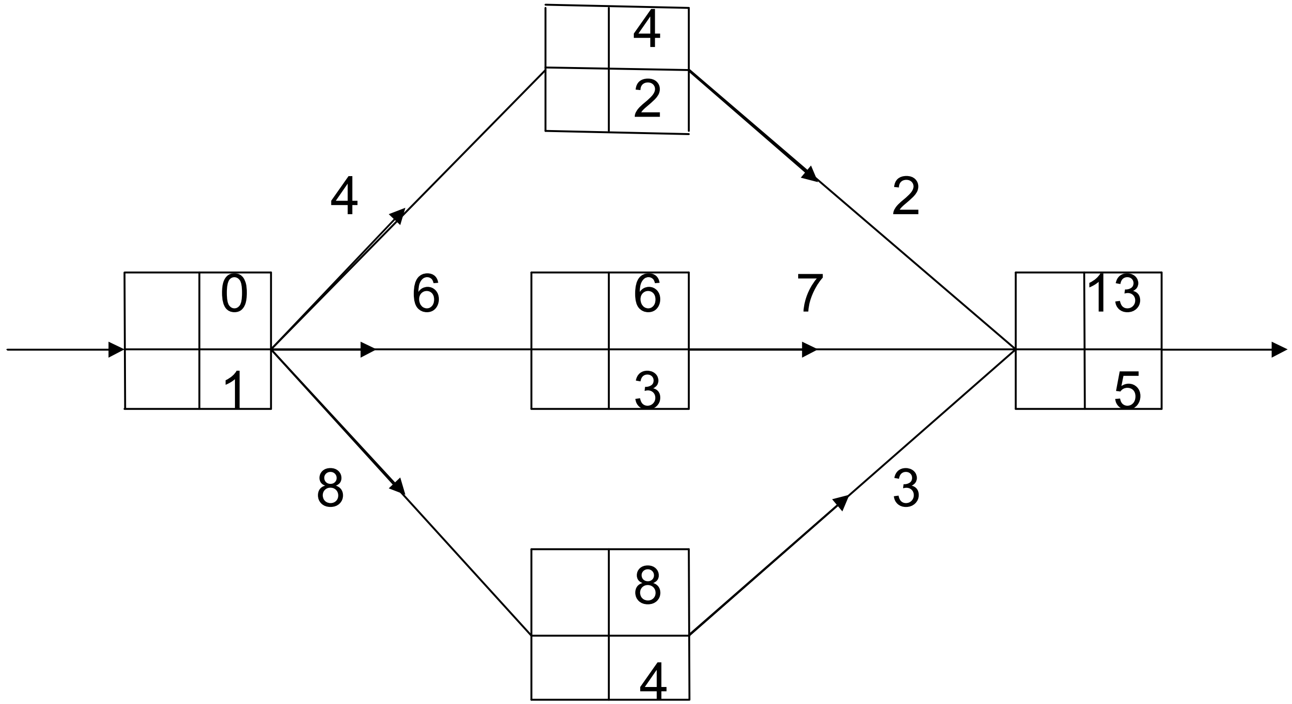
Forward scheduling

Establishes the project start date and then schedules forward from that date.
Based on the planned duration of the required tasks, their interdependencies, and the allocation of resources to complete these tasks, the estimated project completion date is calculated.

Calculation of events' earlier times

$$E(j) = \max_{(i,\ j)} \{E(i) + t(i,j)\},$$

for each event i belonging to the set of the events preceding j and being directly linked with it



Calculation of events' earlier times

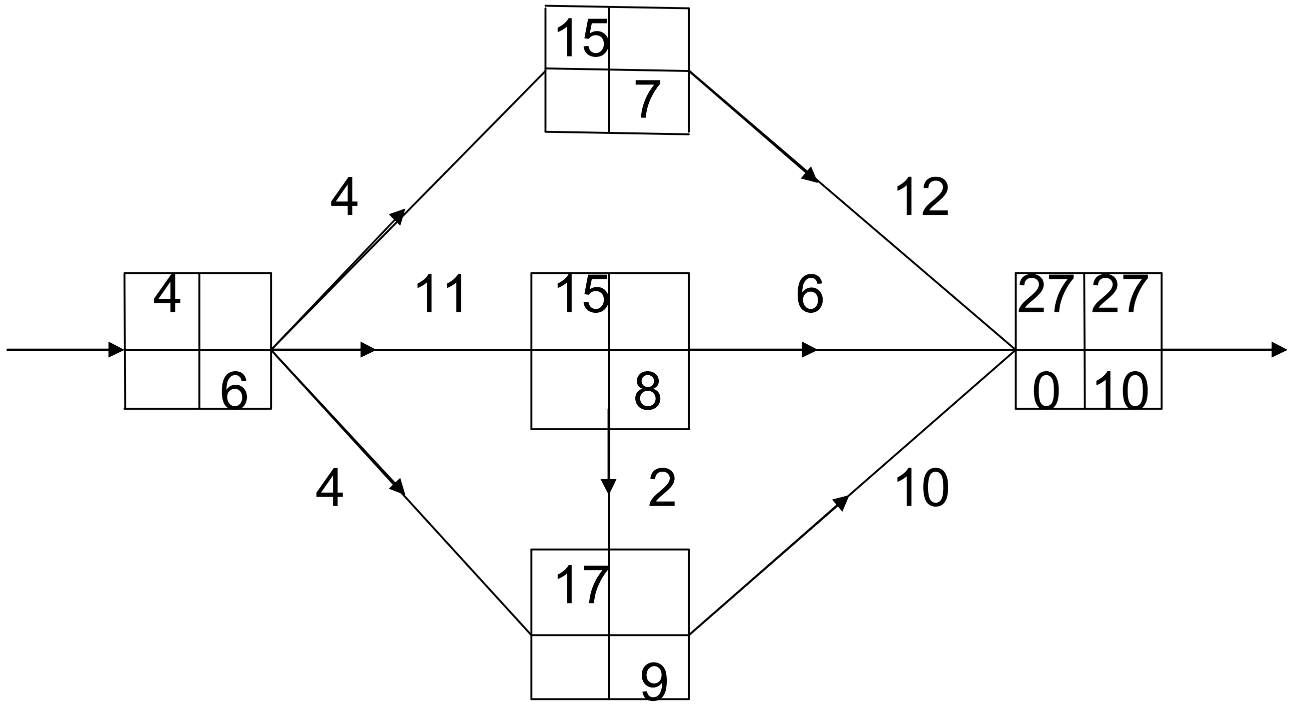
Reverse scheduling

Establishes a project deadline and then schedules backward from that date. Essentially, tasks, their duration, interdependencies, and resources must be considered to ensure that the project can be completed by the deadline set.

Calculation of the latest times of the events

$$L(i) = \min_{(i, j)} \{L(j) - t(i,j)\}$$

for each event j belonging to the set of events succeeding i and being directly linked with it



Calculation of events' latest times

Network solution

For each activity are calculated four time points:

- **ES (Earliest Start)**
  Is the earliest possible moment in time, in which an activity can start.

  ES(i,j) = E(i)

- **EF (Earliest Finish)**
  Is the earliest possible moment in time, in which an activity can be completed.

  EF(i,j) = L(i) + t(i,j)

- **LS (Latest Start)**
  Is the latest possible moment in time, in which an activity can start, without delaying the entire project.

  LS(i,j) = L(j) – t(i,j)

- **LF (Latest Finish)**
  Is the latest possible moment in time, in which an activity can be completed, without delaying the entire project.

  LF(i,j) = L(j)

Activities’ time floats

- **Total Float (TF)**
  Is the maximum possible delay of the beginning of an activity beyond the earliest start time without delaying the entire project.

  $TF(i,j) = L(j) – E(i) – t(i,j)$

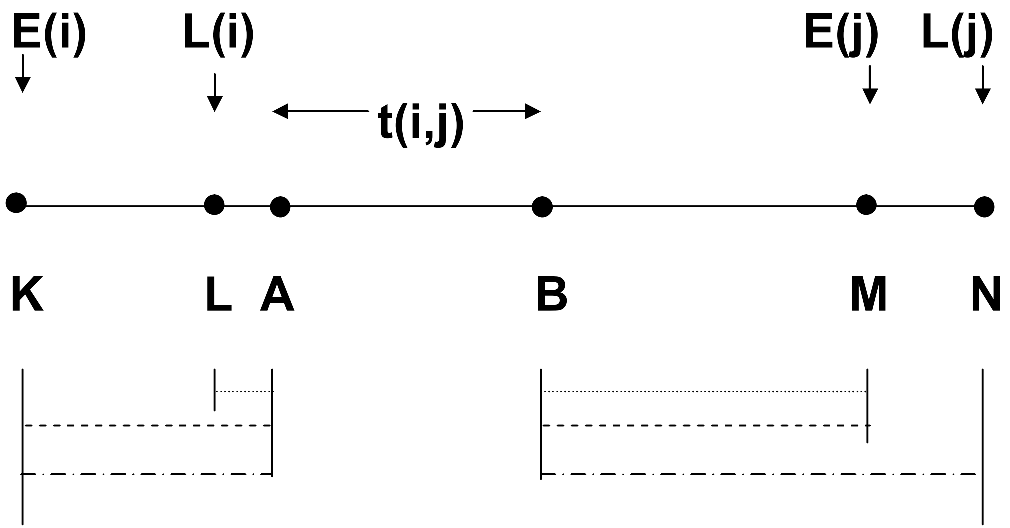
- **Independent Float (IF)**
  Belongs to a specific activity and represents the period of time its duration can be increased without any impact on the preceding or succeeding activities.

  $IF(i, j) = E(j) – L(i) – t(i,j)$

- **Free Float (FF)**
  Is the time allocated for each activity over its duration without any impact on the succeeding activities.

  $FF(i,j) = E(j) – E(i) – t(i,j)$

Activity's time floats



Independent float ( ): IF(i, j) = LA + BM

Free float ( ): FF(i, j) = KA + BM

Total float ( ): TF(i, j) = KA + BN

The **critical path** of a project is the sequence of dependent tasks that have the largest sum of most likely durations. The critical path determines the earliest completion date of the project.

The **slack time** available for any **non critical task** is the amount of delay that can be tolerated between the starting time and completion time of a task without causing a delay in the completion date of the entire project.

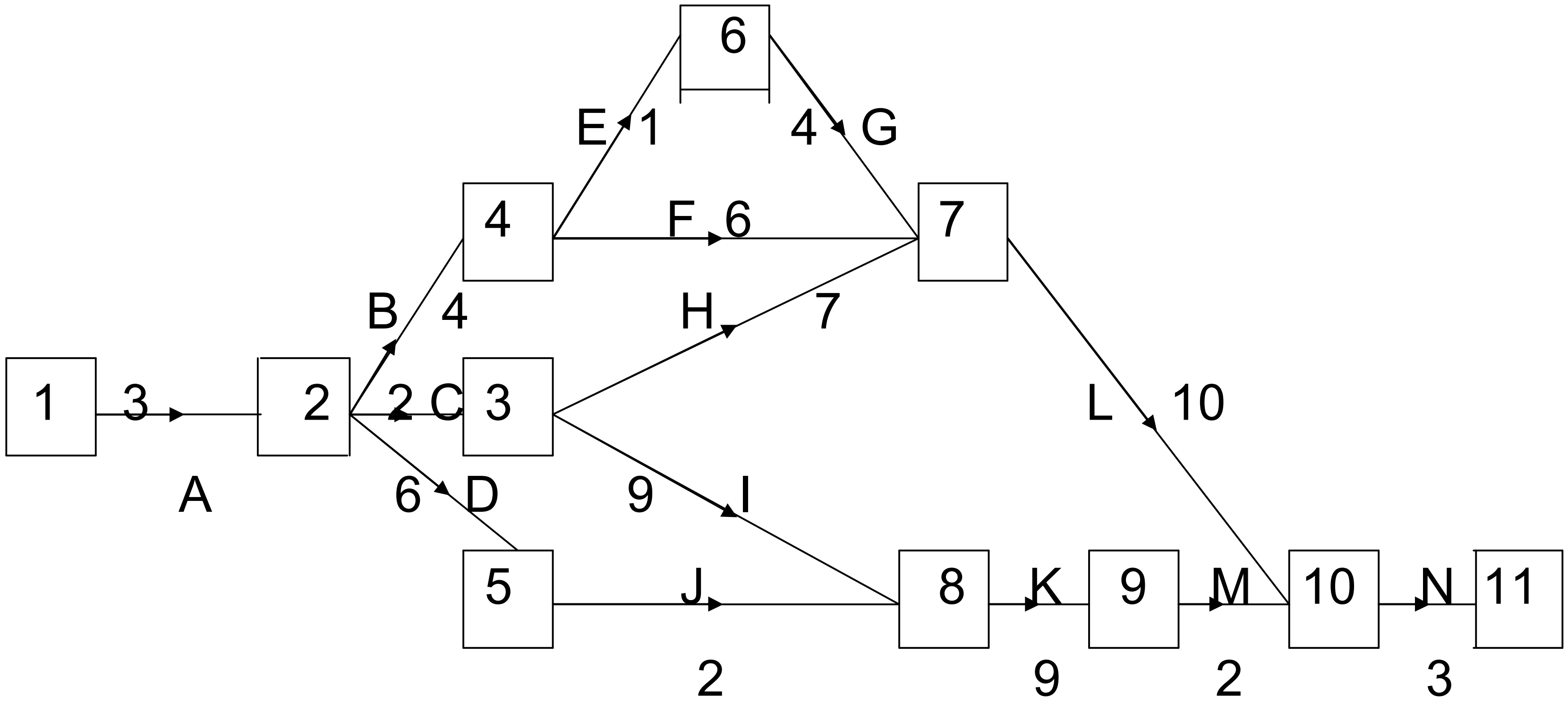
Tasks having positive (> 0) slack time can be delayed to achieve leveling (reduction) of the required resources.

Activities' durations and independencies of an engine assembly project

Activity	A	B	C	D	E	F	G
Duration	3	4	2	6	1	6	4
Preceding activities	-	A	A	A	B	B	E

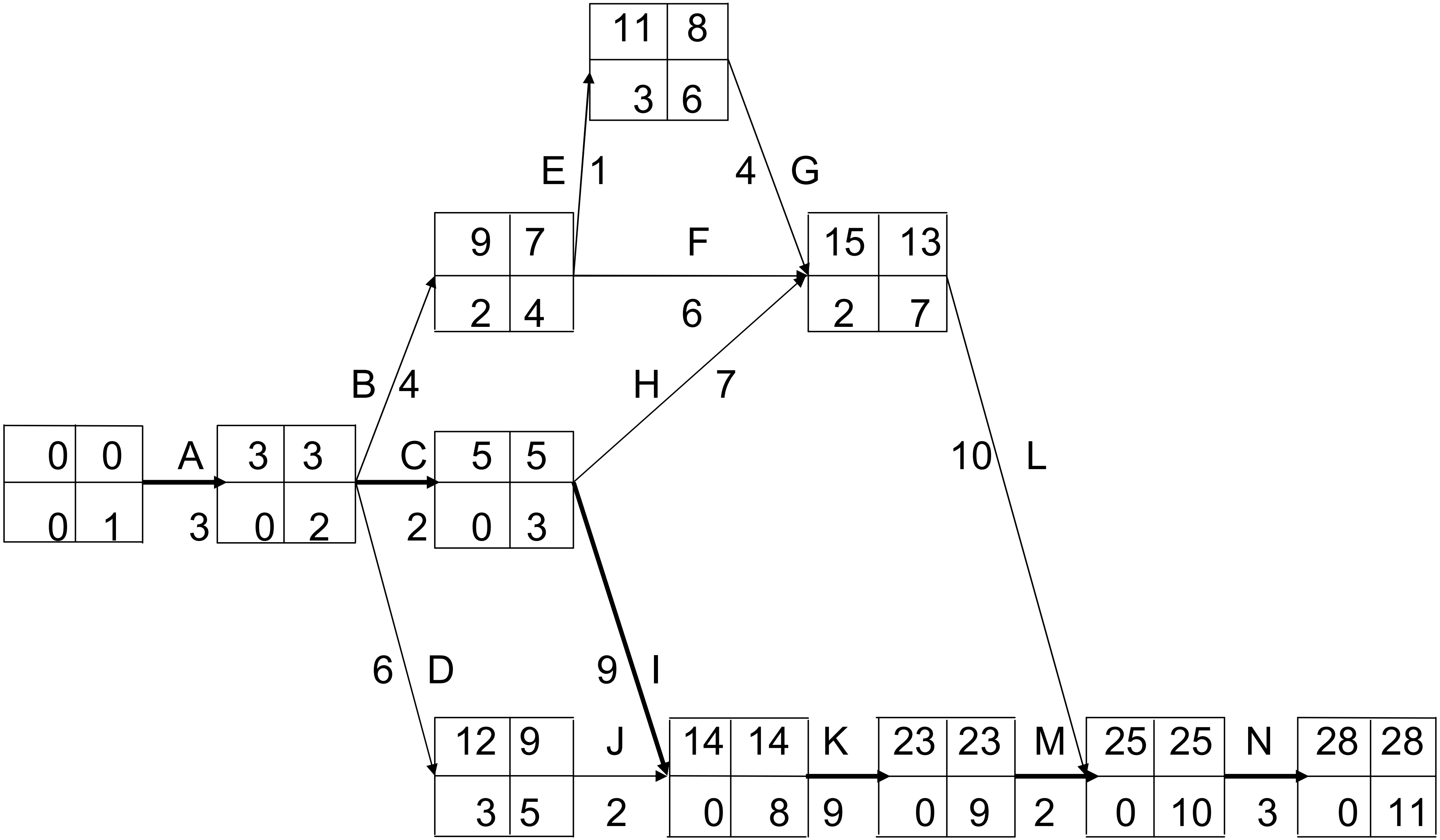
Activity	H	I	J	K	L	M	N
Duration	7	9	2	9	10	2	3
Preceding activities	C	C	D	I,J	F,G,H	K	L,M

Network of an engine assembly project



Activities' times and floats

Activities	Duration t(i,j)	Earliest Times		Latest Times		Time floats		
		ES(I)	EF(j)	LS(i)	LF(j)	IF(i,j)	FF(i,j)	TF(i,j)
A (1,2)	3	0	3	0	3	0	0	0*
B (2,4)	4	3	7	5	9	0	0	2
C (2,3)	2	3	5	3	5	0	0	0*
D (2,5)	6	3	9	6	12	0	0	3
E (4,6)	1	7	8	10	11	-2	0	3
F (4,7)	6	7	13	9	15	-2	0	2
G (6,7)	4	8	12	11	15	-2	1	3
H (3,7)	7	5	12	8	15	1	1	3
I (3,8)	9	5	14	5	14	0	0	0*
J (5,8)	2	9	11	12	14	0	3	3
K (8,9)	9	14	23	14	23	0	0	0*
L (7,10)	10	13	23	15	25	0	2	2
M (9,10)	2	23	25	23	25	0	0	0*
N (10,11)	3	25	28	25	28	0	0	0*



Network solution of engine assembly project

Importance of slack time and critical path

- Slack (or float) shows how much allowance each activity has, i.e. how long it can be delayed without affecting the completion date of the whole project.

- Critical path is a sequence of activities from start to finish with zero slack.
  Critical activities are the activities on the critical path.

- Critical path identifies the minimum time to complete the project.

- If any activity on the critical path is shortened or extended, the duration of the project will be shortened or extended accordingly.

- A significant effort should be put in trying to control all the critical activities, so that the project can meet the planned completion date. If the duration of any critical activity is lengthened, be aware that the project will not meet the deadline and some action needs to be taken.

- If extra resources can be spent to speed up some activities, do so only for critical activities.

- Do not waste resources on non-critical activities, as it will not shorten the duration of the project.

- If resources can be saved by lengthening the duration of some activities, do so for non-critical activities up to the limit of their total float.

Planning projects under uncertainty

PERT technique

(Project Evaluation Review Technique)

PERT technique was developed by the US Navy for the planning and control of the Polaris missile program and the emphasis was on completing the program in the shortest possible time. In addition PERT had the ability to cope with uncertain activity completion times.

It is used in Project Management for non-repetitive jobs (research and development work), where the time and cost estimates tend to be quite uncertain.

PERT technique uses probabilistic time estimates.

Features of PERT technique

- The durations of the activities cannot be determined with relative accuracy.

- The durations are usually estimates and are subject to variance.

- Probabilistic models are used to estimate the expected required time to complete the project.

- Examines questions such as:
  - Which is the average duration of the project?
  - Which is the maximum possible duration of the project?
  - Which is the probability of project completion within a certain given time?
  - Which is the completion time with a certain given probability?

For each activity with uncertain duration three time estimates are used:

- **Optimistic time (a)**
  The lowest time that is expected to last the activity (ideal conditions)

- **Most likely time (m)**
  The most frequent duration of the activity (normal conditions)

- **Pessimistic time (b)**
  The largest time that is expected to last the activity (worst possible conditions)

It is considered that the activities with uncertain duration follow the Beta probability distribution.

Expected duration of the activity (t_e)

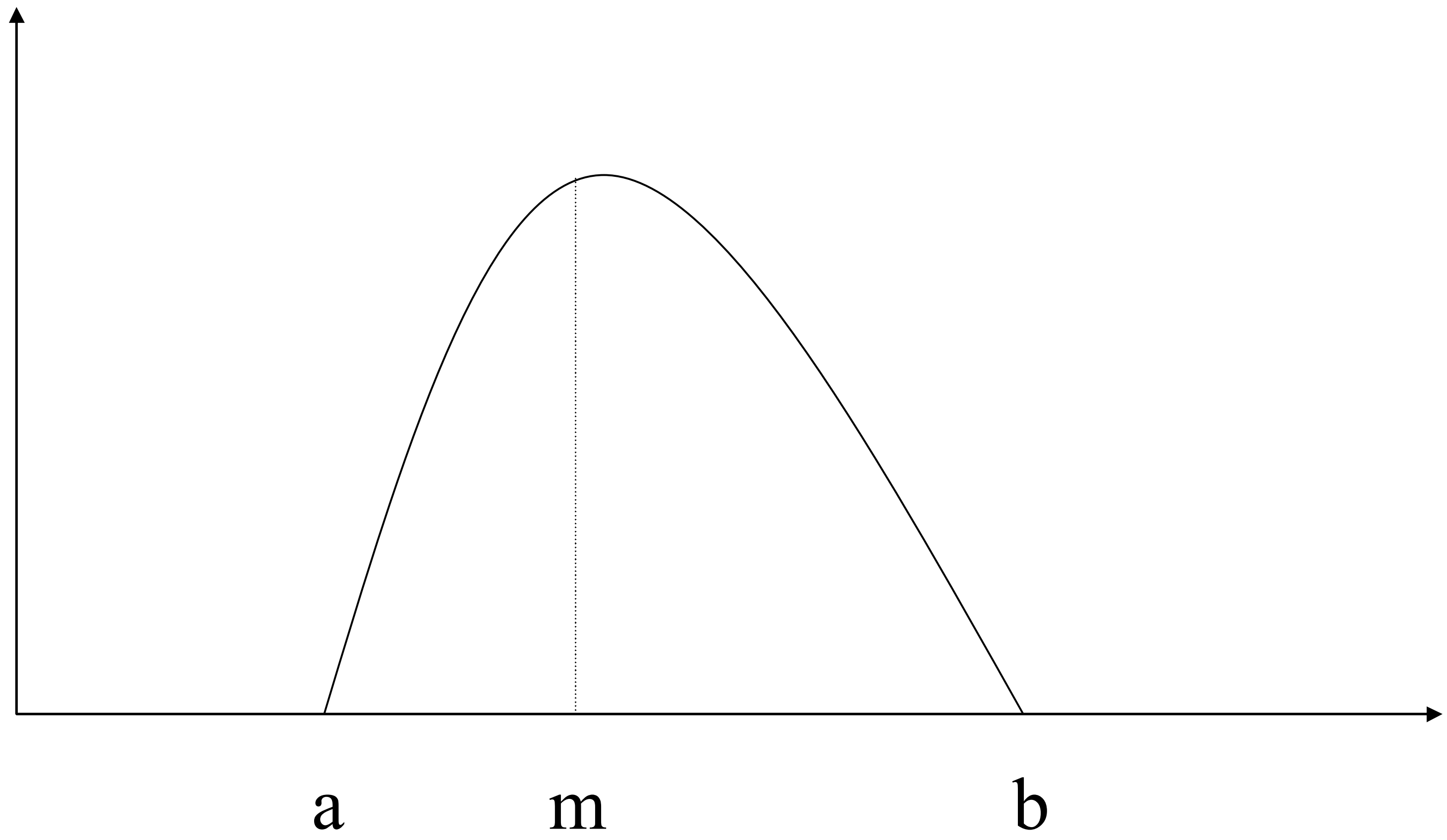
$$t_e = \frac{a + 4m + b}{6}$$

Activity's variance (σ^2)

$$\sigma^2 = \left(\frac{b-a}{6}\right)^2 = \frac{(b-a)^2}{36}$$

Activity's standard deviation (σ)

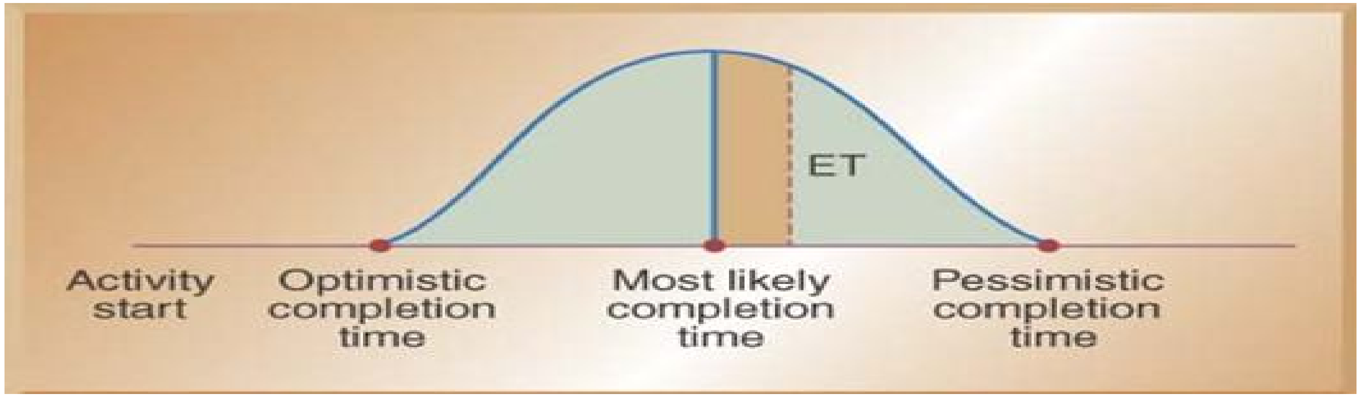
$$\sigma = \frac{b - a}{6}$$



Probability density function of Beta distribution

Using Beta Probability Distribution to Calculate Expected Time Durations

- **A typical beta distribution is shown below, note that it has definite end points**
- **The expected time for finishing each activity is a weighted average**



$$\text{Exp. time} = \frac{\text{optimistic} + 4(\text{most likely}) + \text{pessimistic}}{6}$$

Average duration of the project (μ)

The estimation of the expected duration of the project is the sum of the expected durations of the activities on the critical path

Variance of project duration (σ^2_{CP})

The variance of project duration, reflecting the degree of its uncertainty, is the sum of the variances of the critical activities

$$\sigma^2_{CP} = \sum_{activity \in CP} \sigma^2$$

Standard deviation of project duration (σ_{CP})

Equals to the square root of the variance of project duration

$$\sigma_{CP} = \sqrt{\sum_{activity \in CP} \sigma^2}$$

1st example of PERT technique application

Activity	a	m	b
A	10	15	20
B	6	10	15
C	2	5	10
D	11	18	20
E	1	3	5
F	4	8	10
G	7	10	13
H	6	12	15
I	4	7	10

Calculations

$$t_A = \frac{10 + 4 \times 15 + 20}{6} = \frac{90}{6} = 15$$

$$\sigma_{\mathrm{A}}^2 = \frac{(20 - 10)^2}{36} = \frac{100}{36} = 2.7778$$

$$t_B = \frac{6 + 4 \times 10 + 15}{6} = \frac{61}{6} = 10.1667$$

$$\sigma_B^2 = \frac{(15 - 6)^2}{36} = \frac{81}{36} = 2.25$$

$$t_C = \frac{2 + 4 \times 5 + 10}{6} = \frac{32}{6} = 5.3334$$

$$\sigma_C^2 = \frac{(10 - 2)^2}{36} = \frac{64}{36} = 1.778$$

$$t_D = \frac{11 + 4 \times 18 + 20}{6} = \frac{103}{6} = 17.1667$$

$$\sigma_D^2 = \frac{(20 - 11)^2}{36} = \frac{81}{36} = 2.25$$

$$t_I = \frac{4 + 4 \times 7 + 10}{6} = \frac{42}{6} = 7$$

$$\sigma_I^2 = \frac{(10 - 4)^2}{36} = \frac{36}{36} = 1$$

Results from computer (QSB+)

| Expected Completion Time = 70.83334 CPU seconds = 0 |

| Total Variance = 14.75 Total Variance on Critical Path = 11.52778 |

|--|

Critical path: A ==> B ==> D ==> G ==> Dummy2 ==> H ==> I.

Variance on this path = 11.52778 (Std Dev = 3.395)

Expected project duration = $t_A + t_B + t_D + t_G + t_H + t_I = 70.83$

Variance $= \sigma^2_A + \sigma^2_B + \sigma^2_D + \sigma^2_G + \sigma^2_H + \sigma^2_I = 11.527$

Standard deviation= $\sqrt{\sigma^2_A + \sigma^2_B + \sigma^2_D + \sigma^2_G + \sigma^2_H + \sigma^2_I} = 3.395$

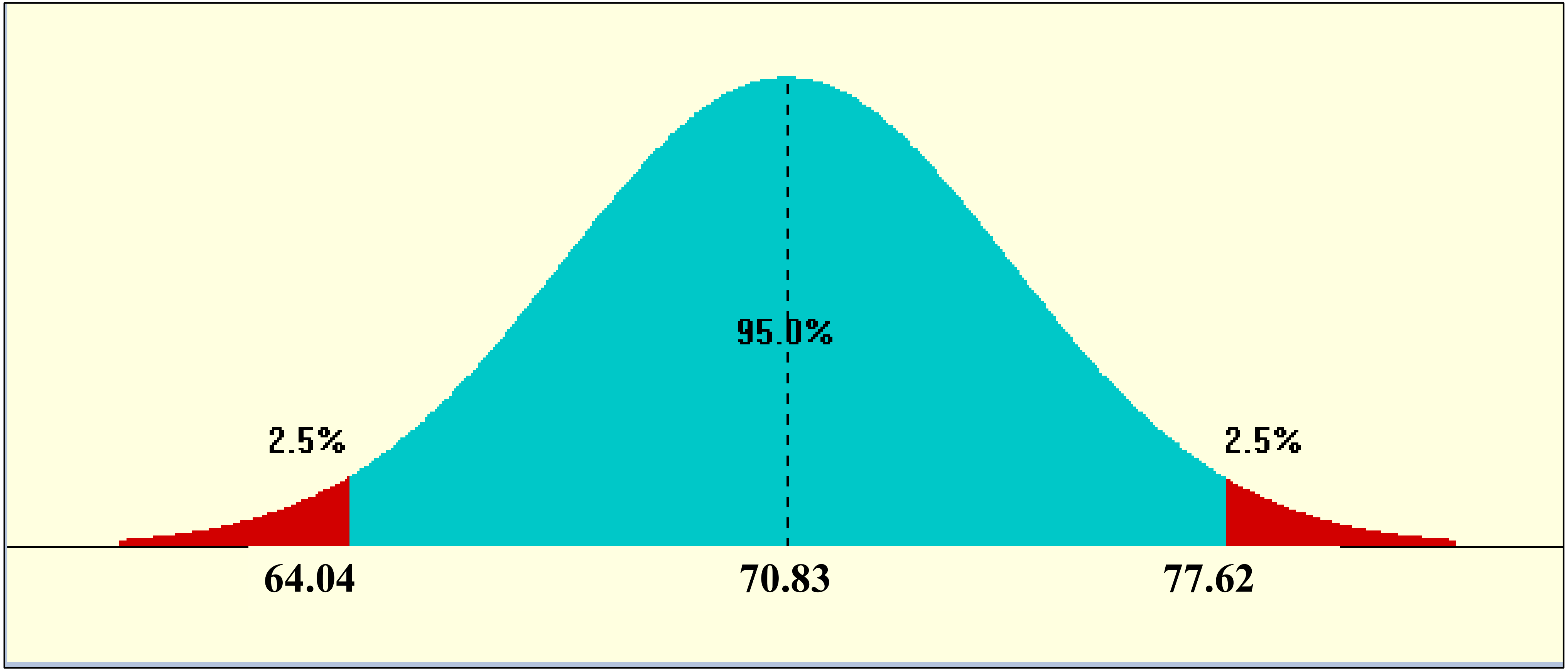
This normal distribution **N (μ, σ^2)** has parameters:

- **μ** = the sum of the expected times of critical activities
- **σ^2** = the sum of the variances of the critical activities

Use of normal distribution

At the example: μ=70.83 and σ=3.395 → N(70.83, 3.395^2)

- 99% of the values are in the range μ±3σ ➔ (60.64, 81.01)
- 95% of the values are in the range μ±2σ ➔ (64.04, 77.62)
- 68% of the values are in the range μ±σ ➔ (67.43, 74.22)



The random variable $Z = \dfrac{x - \mu}{\sigma}$

Where:

Z the random variable of normal distribution

x the given or desired duration of the project

μ the expected duration of the project

σ the standard deviation of project duration

It follows the typical normal distribution N (0, 1) so:

Possibility (project duration < x) = Possibility ($Z < z_x$)

Normal distribution curve

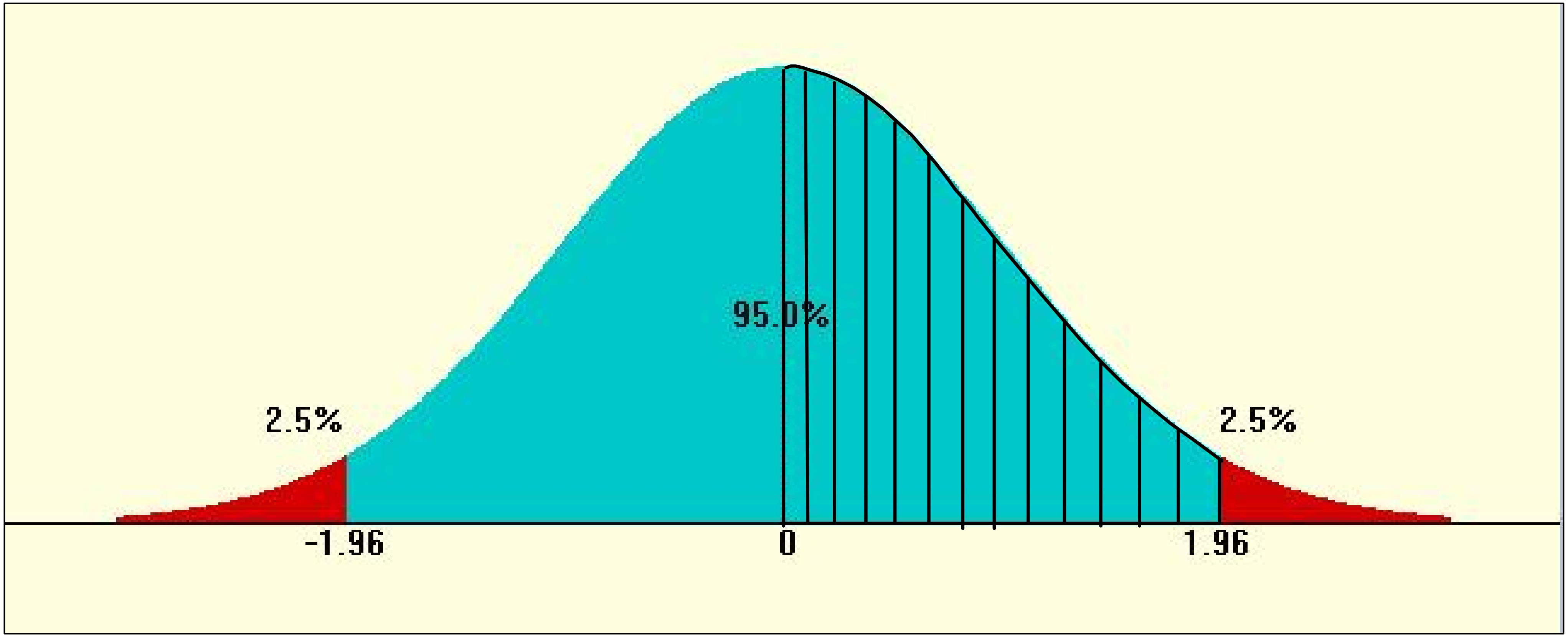


Table of standard normal distribution Z ~ N(0,1)

ΠΙΝΑΚΑΣ ΤΥΠΙΚΗΣ ΚΑΝΟΝΙΚΗΣ ΚΑΤΑΝΟΜΗΣ

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4986	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Using the formula of normal distribution

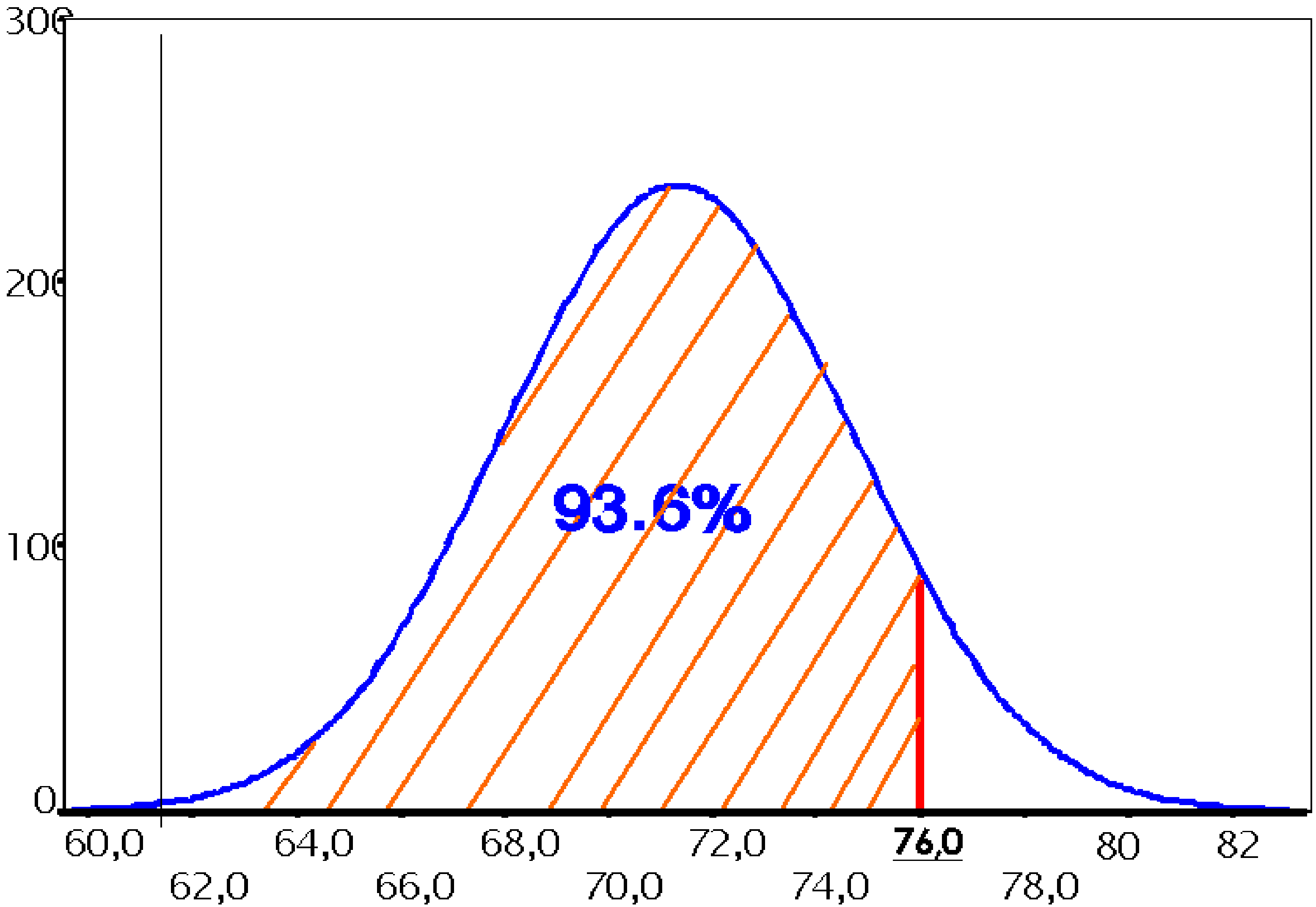
The possibility of the project to be completed in time up to 70.83 days (average project duration) is 50%.

The possibility to be completed in up to 76 days is for x=76:

$$Z_{76} = \frac{76 - 70.83}{3.395} = 1.523$$

According to the tables of normal distribution, the value Z = 1.523 corresponds to a probability 0.9359, i.e. the probability of the project to be completed in 76 days is about 93.6%.

Corresponding normal distribution chart



3. The probability of the project to be completed in up to 66 days is for x = 66

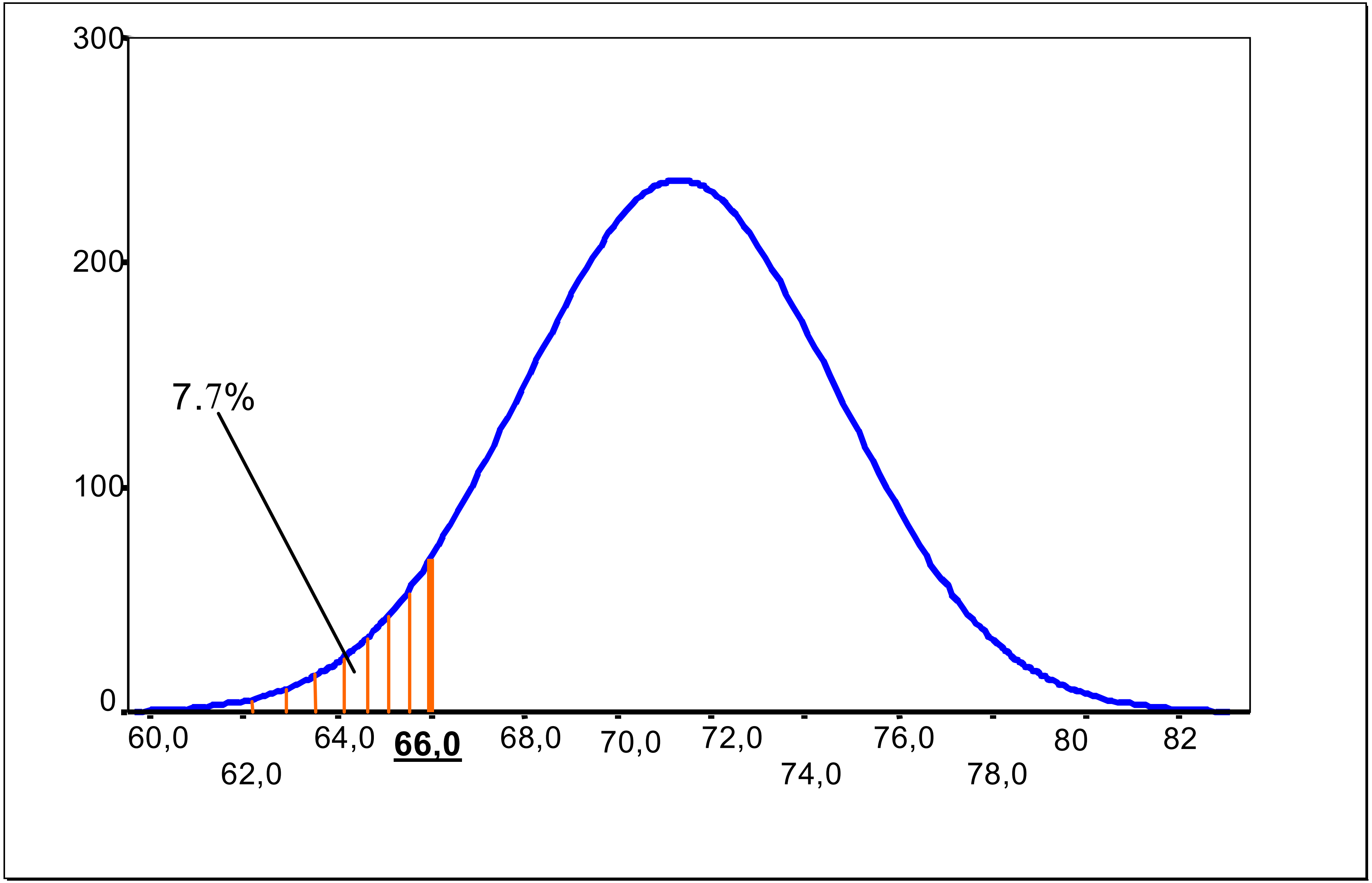
$$\text{Prob}(Z < \frac{66 - 70.83}{3.395}) = \text{Prob}(Z < -1.4226)$$

i.e.

Probability (Z > 1.4226) = 1 - Probability (Z < 1.4226) = 1 - 0.9223 = 0.0777

So, the probability of the project to be completed in time maximum up to 66 days is about 7.8%.

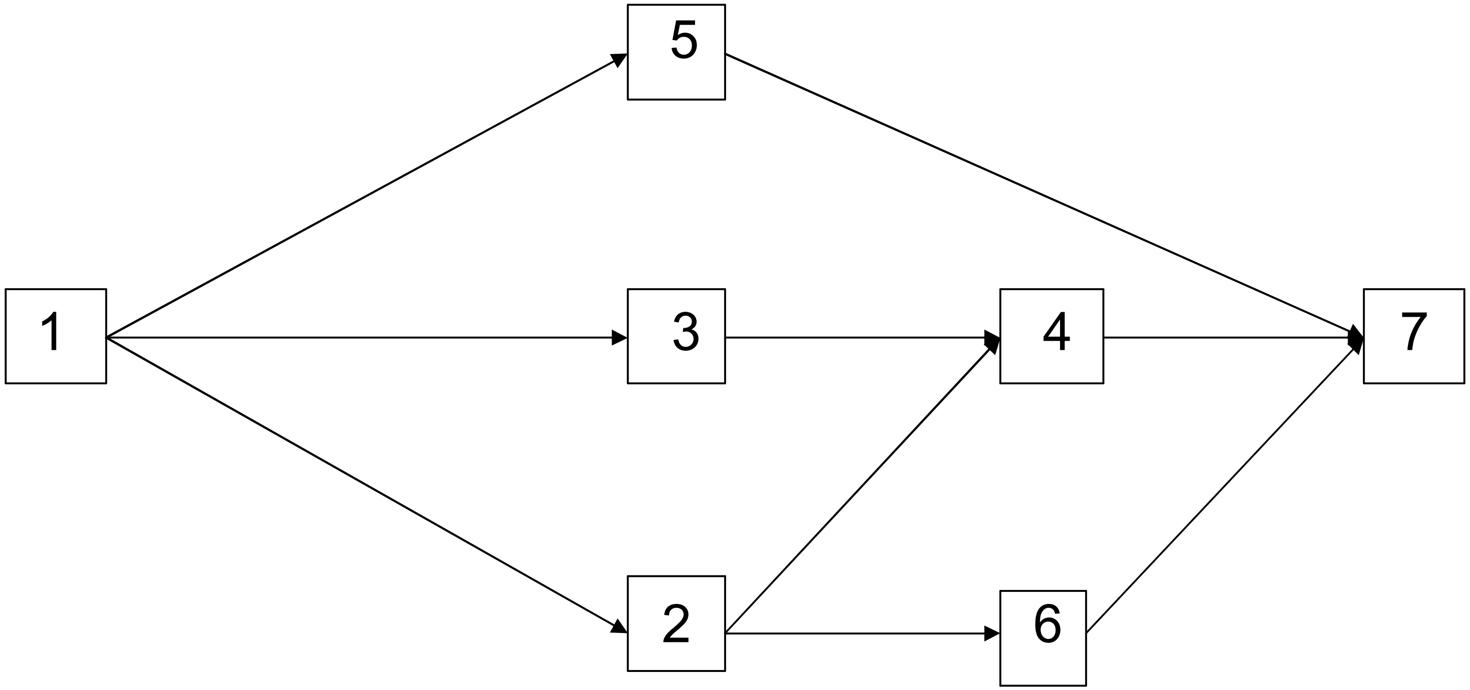
Corresponding normal distribution diagram



Calculate:

- The probability of the project to be completed in up to 55 days
- The probability of the project to be completed in 10 to 11 weeks

2nd example of PERT technique application



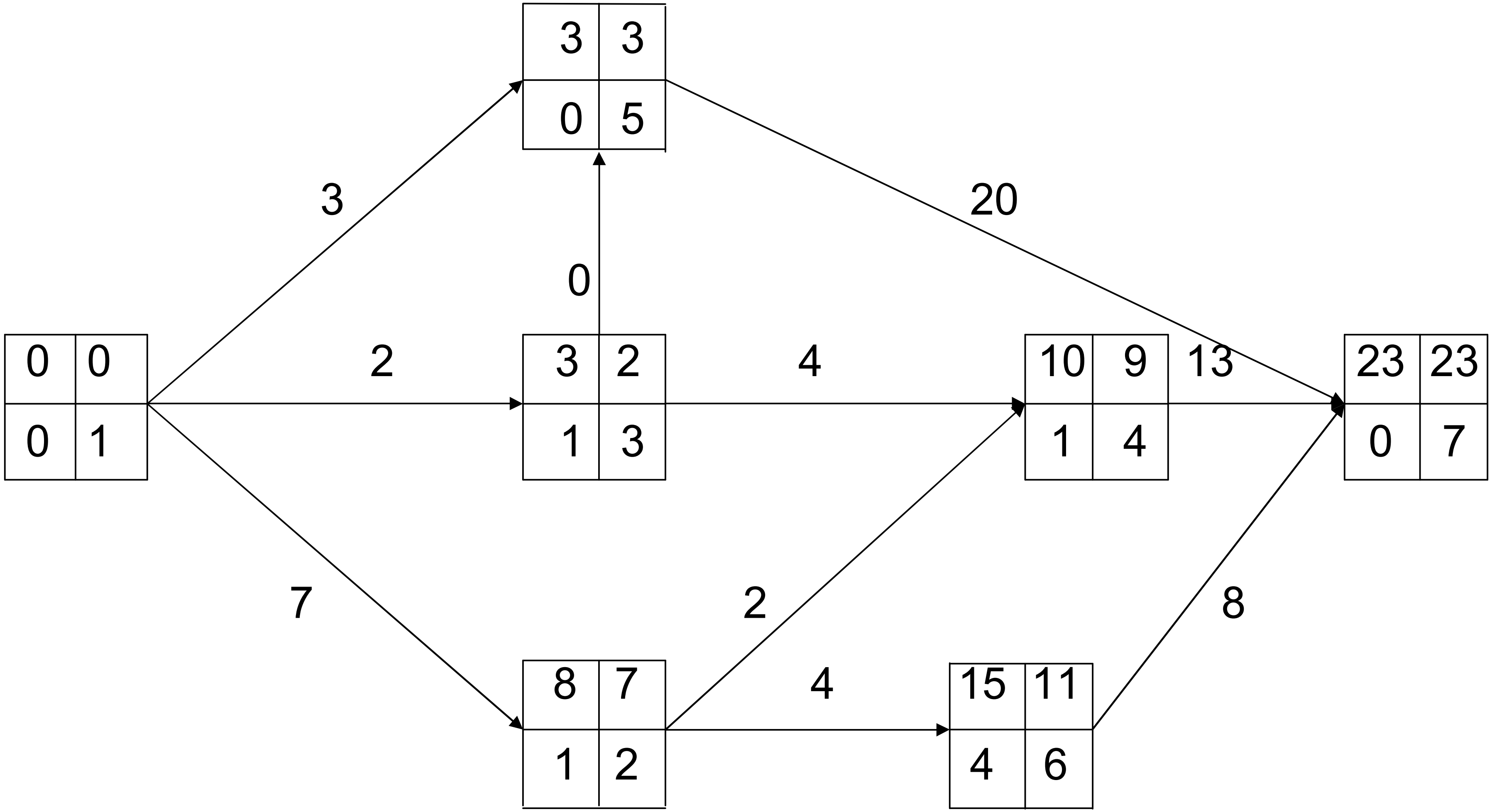
Network of a bottling project

Estimates of activities' durations

Activity (i,j)	Optimistic estimation (a)	Most likely estimation (m)	Pessimistic estimation (b)
1-2	3	6	15
1-3	0.5	1	7.5
1-5	1	2	9
2-4	0.5	1	7.5
2-6	2	3	10
3-4	1	3	11
3-5	0	0	0
4-7	6	10	32
5-7	10	15	50
6-7	3	5	25

Calculations of activities' times

Activity (i,j)	Durations a	m	b	Expected duration $t_e(i,j)$	Standard deviation σ(i,j)	Total float
1-2	3	6	15	7	2	1
1-3	0,5	1	7,5	2	7/6	1
1-5	1	2	9	3	4/3	0*
2-4	0,5	1	7,5	2	7/6	1
2-6	2	3	10	4	4/3	4
3-4	1	3	11	4	5/3	4
3-5	0	0	0	0	0	1
4-7	6	10	32	13	4 και 1/3	1
5-7	10	15	50	20	6 και 2/3	0*
6-7	3	5	25	8	3 και 2/3	4



Solution of the bottling project network

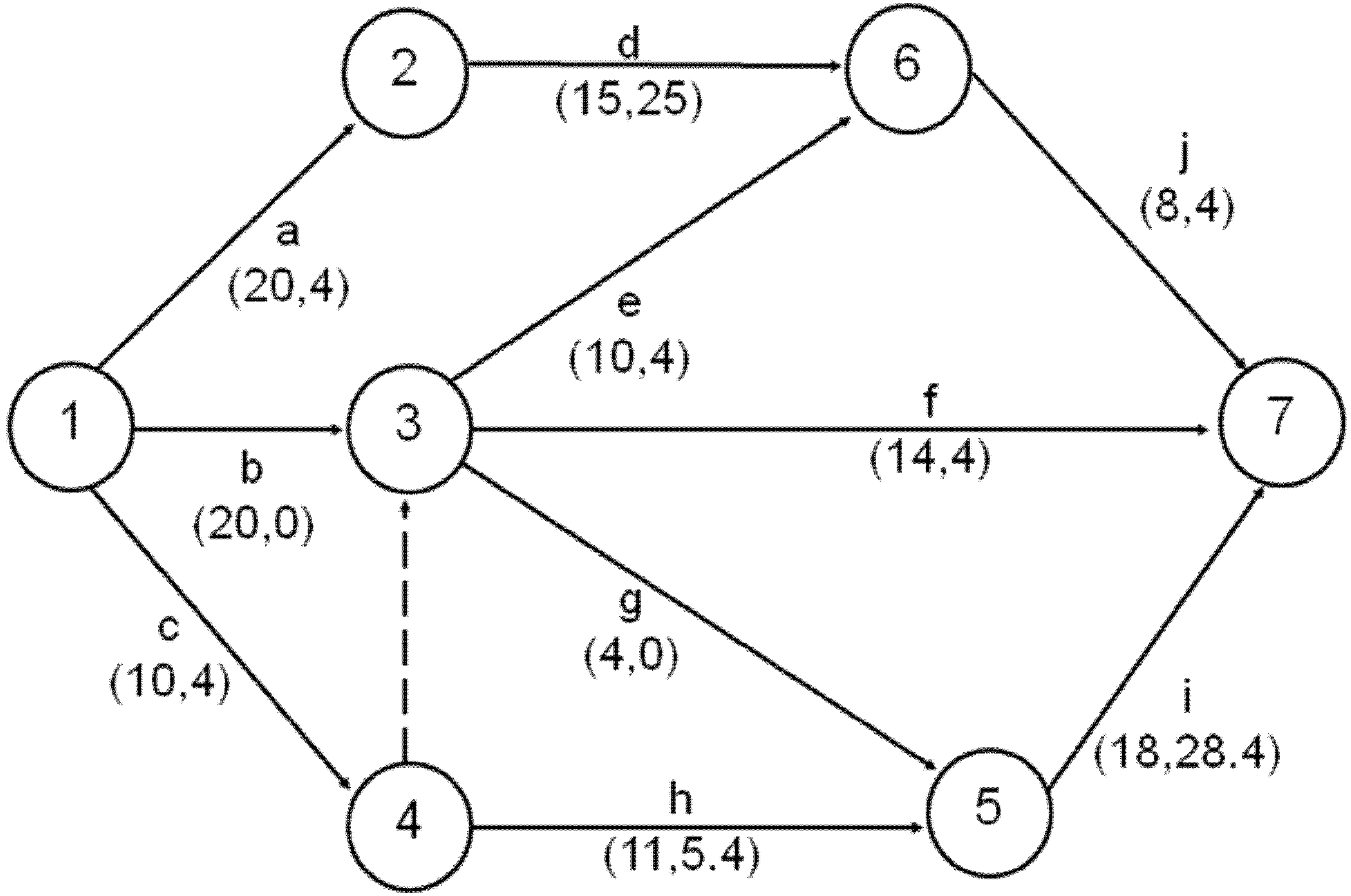
Calculate:

- The average duration and the standard deviation of the project
- The probabilities of the project to be completed in 20, 29 and 32 days respectively
- The required duration of the project to be completed with probability 10%, 60% and 95% respectively
- Design a T – p (duration – possibility of project completion) diagram of 4 points (t_i, p_i), i = 1, 2, … 4 with coordinates of your choice

3^{rd} example of PERT technique application
Activities' predecessors and times

Activity	Immediate Predecessor	Optimistic Time	Most likely Time	Pessimistic Time	Expected duration	Variance	Standard deviation
a	-	10	22	22	20	4	2
b	-	20	20	20	20	0	0
c	-	4	10	16	10	4	2
d	a	2	14	32	15	25	5
e	b, c	8	8	20	10	4	2
f	b, c	8	14	20	14	4	2
g	b, c	4	4	4	4	0	0
h	C	2	12	16	11	5.4	2.32
i	g, h	6	16	38	18	28.4	5.33
j	d, e	2	8	14	8	4	2

The complete network



Activities' main times

Activity	ES	LS	Float	Critical?
a	0	0	0	Yes
b	0	1	1	
c	0	4	4	
d	20	20	0	Yes
e	20	25	5	
f	20	29	9	
g	20	21	1	
h	10	14	4	
i	24	25	1	
j	35	35	0	Yes

a) Assume that the project manager promised to complete the project in fifty days. Which are the chances of meeting that deadline?

Calculate Z, where

$$Z = \frac{x - \mu}{\sigma}$$

x = 50; μ (scheduled date) = 20+15+8 = 43; σ^2 = 4+25+4 = 33

Z = (50 – 43) / 5.745 = 1.22 standard deviations.

The probability value of Z = 1.22, is 0.888

Thus, the probability of meeting the deadline of fifty days is 88.8%

b) What deadline are you 95% sure of meeting?

Z value associated with 0.95 is 1.645

$x = \mu + Z * \sigma = 43 + 5.745 * 1.645 = 43 + 9.45 = 52.45$ days

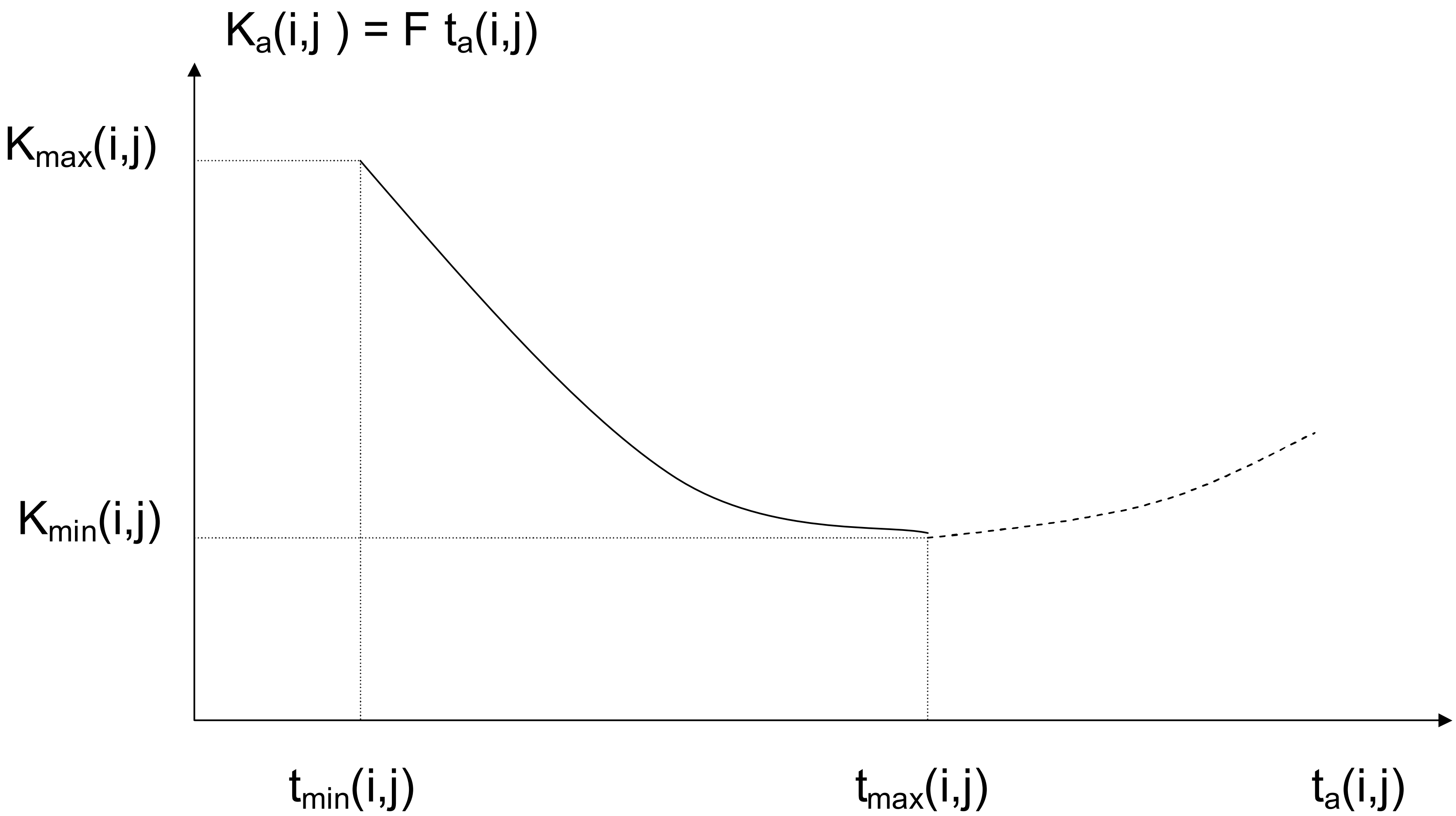
Thus, there is a 95 percent chance of finishing the project by 52.45 days.

Minimization of project cost

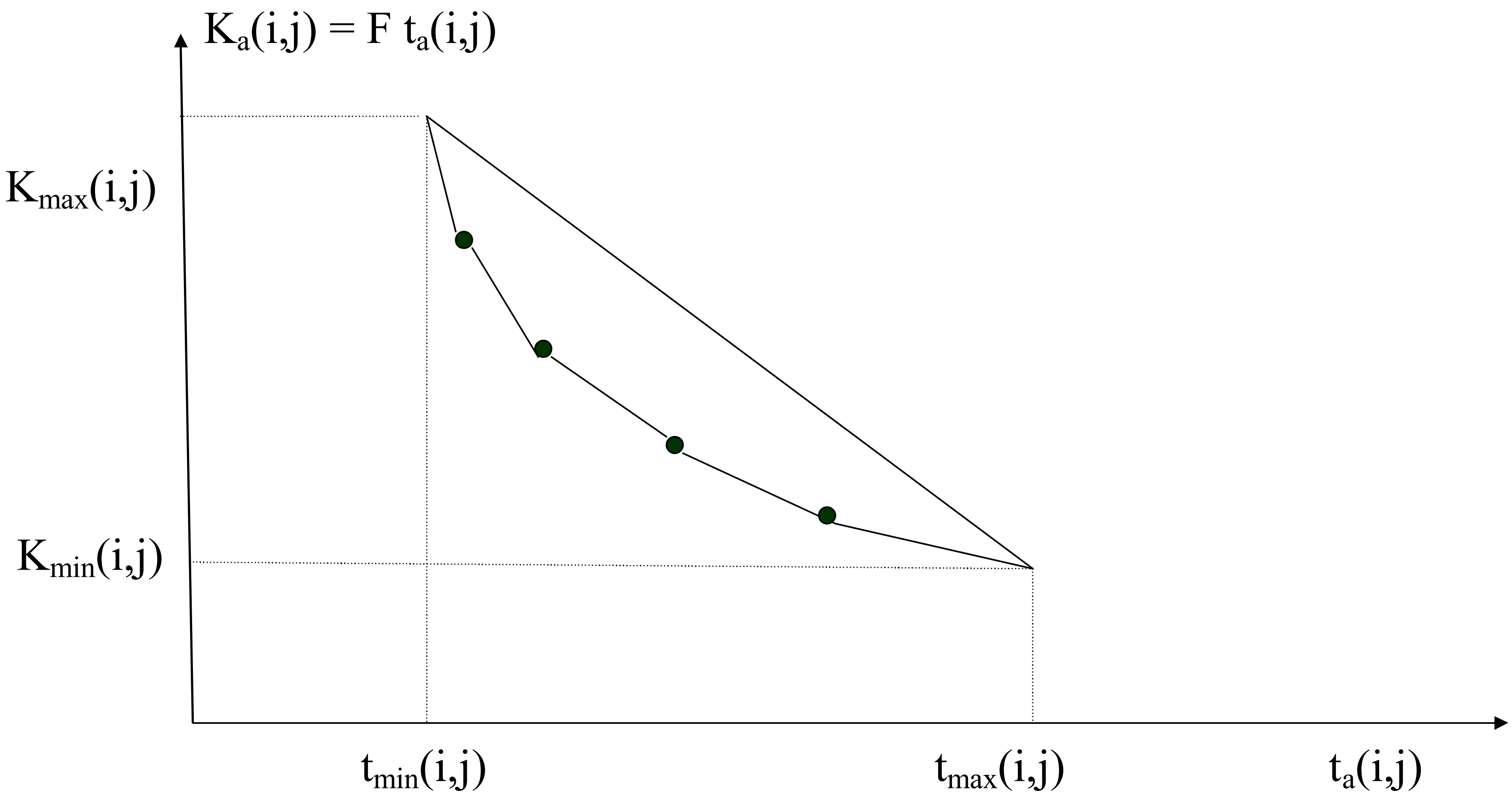
CPM

(Critical Path Method)

CPM was developed by Du Pont and the emphasis was on the trade-off between the cost of the project and its overall completion time (e.g. for certain activities it may be possible to decrease their completion times by spending more money - how does this affect the overall completion time of the project)



Cost – duration function of an activity



Approximate cost – duration function of the activity

Cost coefficient of an activity - c(i, j)

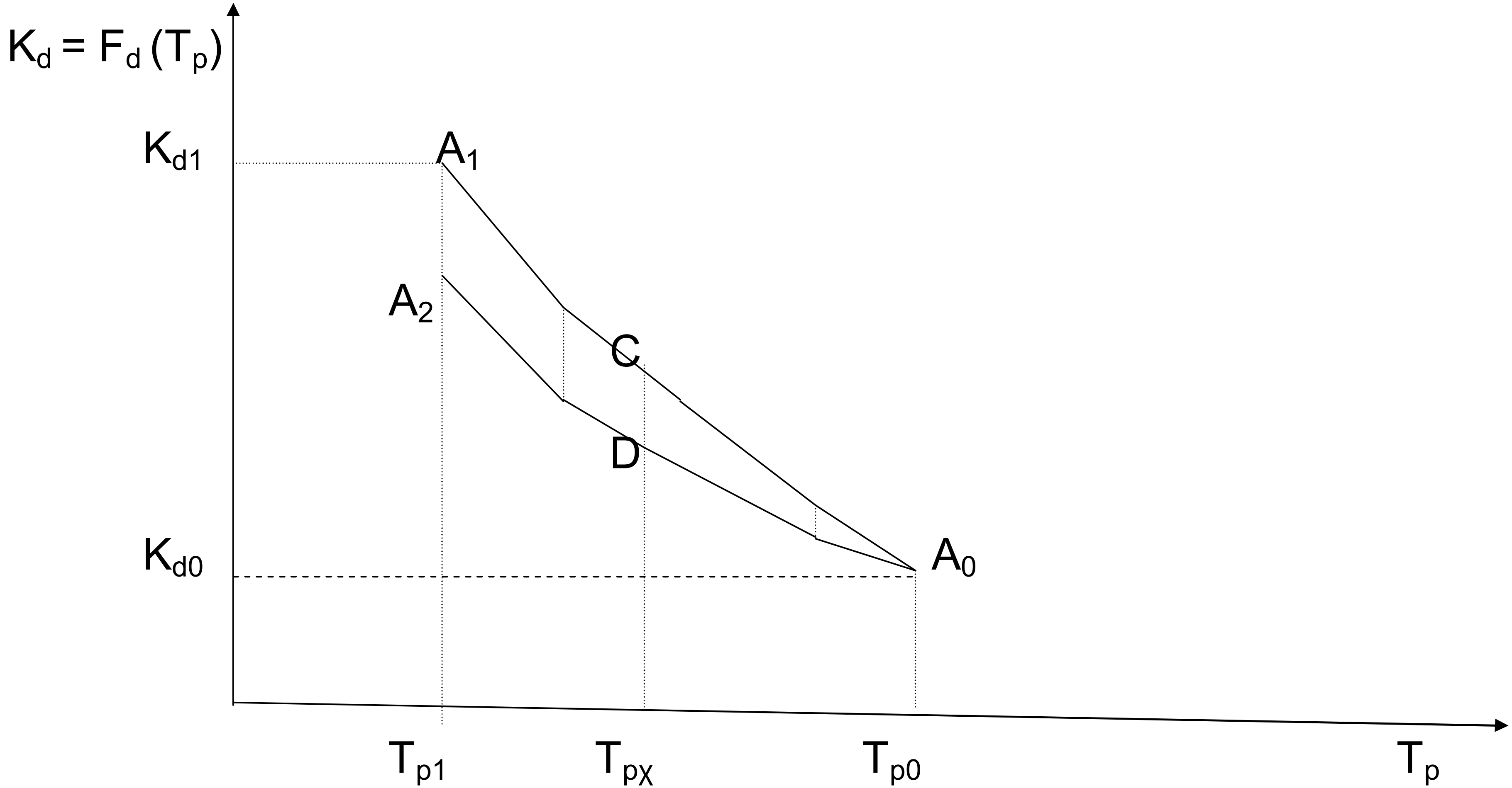
$$c(i,j)=\frac{K_{\max}(i,j)-K_{\min}(i,j)}{t_{\max}(i,j)-t_{\min}(i,j)}$$

c(i,j) is expressed in monetary units per time unit (e.g. €/day)

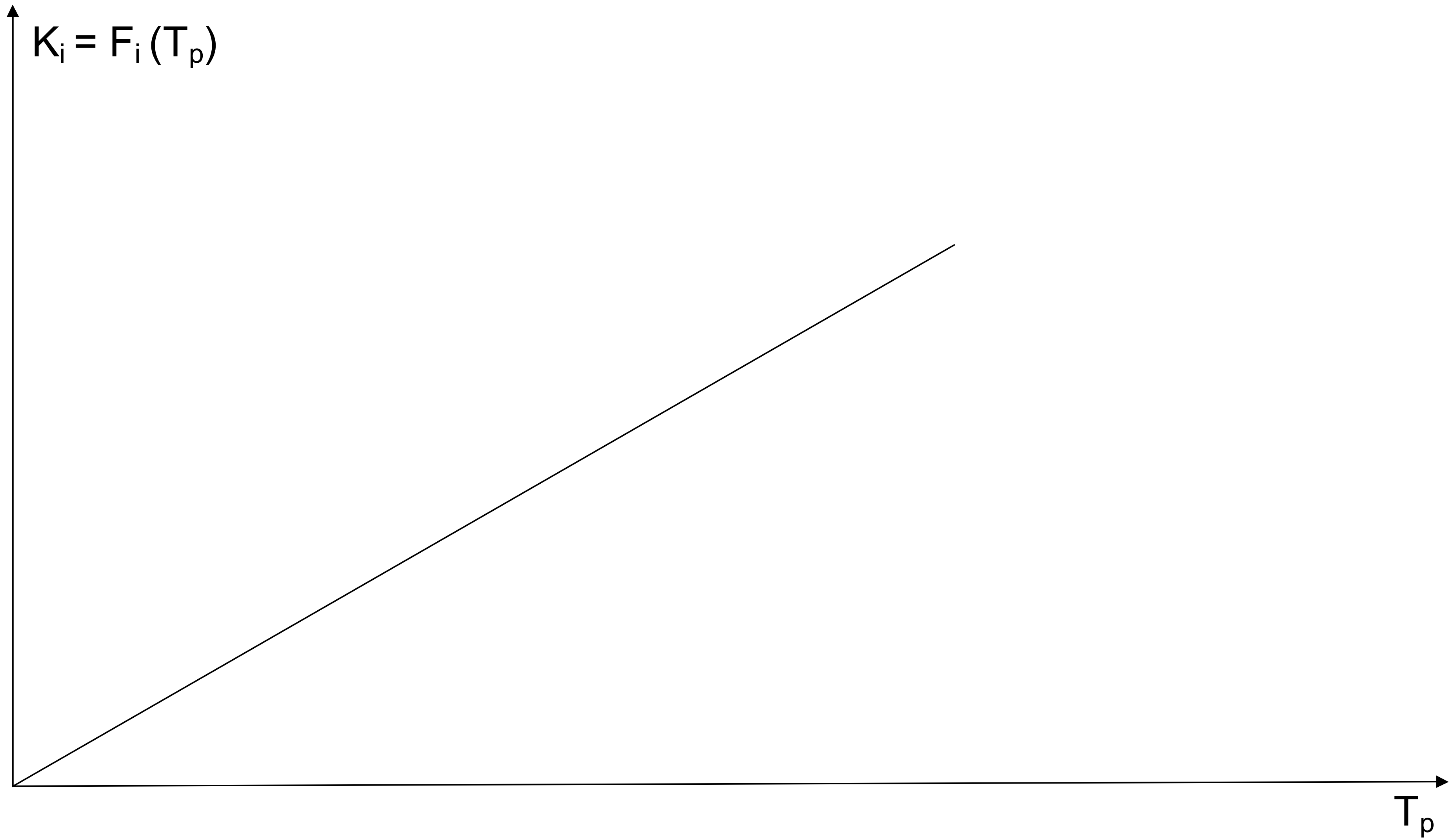
Indicates the amount of cost increase (or decrease) of the activity per time unit of increase (or decrease) of the duration, between the duration limits t_{min} and t_{max}.

In order to reduce the duration of the project by one time unit, the duration of the critical activity with the lowest cost coefficient must be reduced by one time unit.

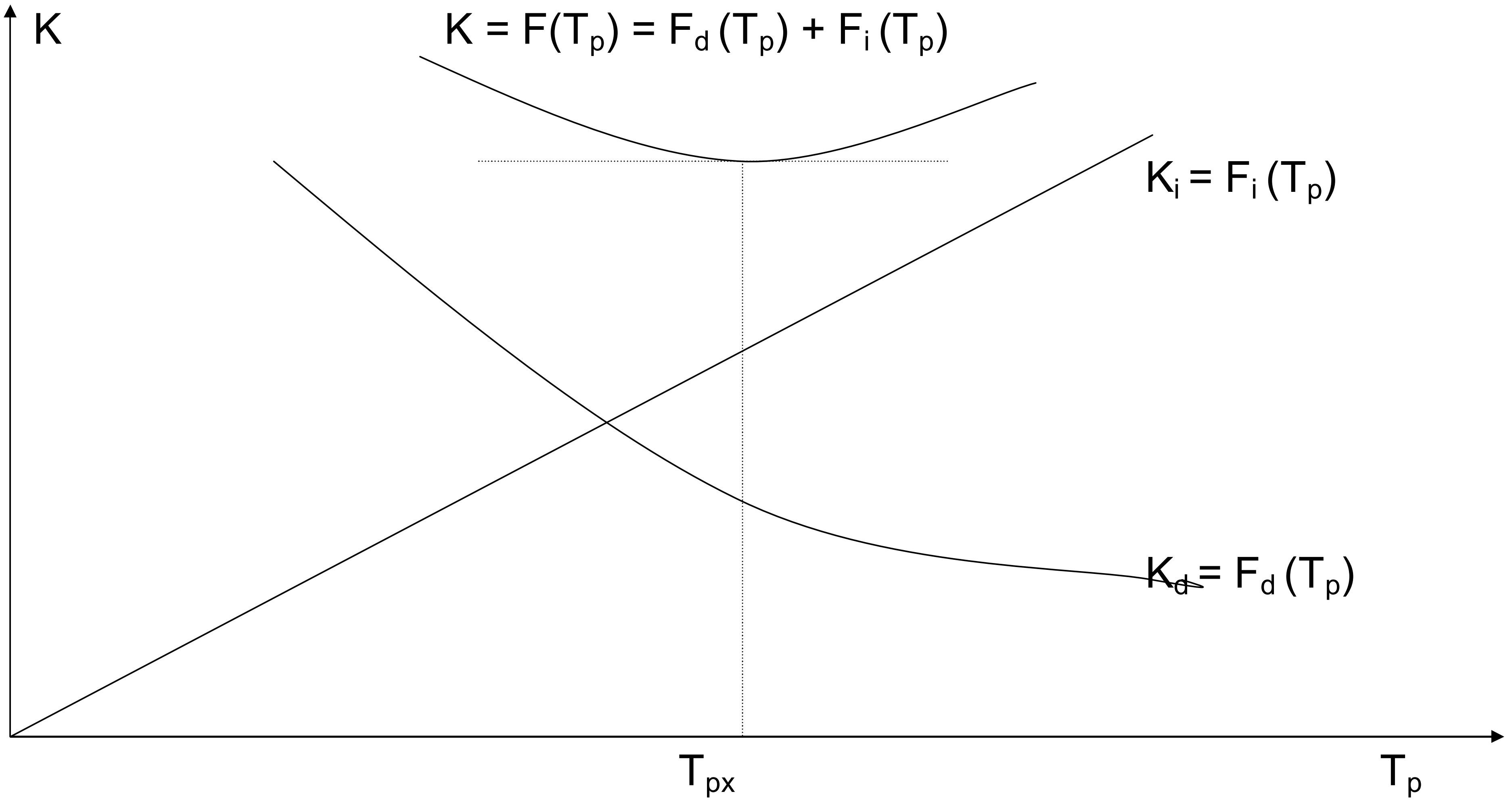
When there is more than one critical path, the duration of one activity of every critical path must be reduced. The combination of the activities with the minimum sum of the respective cost coefficients must be selected.



Function of direct cost - total duration of the project



Function of indirect cost – total duration of the project



Function of indirect cost – total duration of the project

Methodology of the CPM method

1) Solution of the network with the minimum durations of the activities which correspond to their maximum direct cost. Determination of critical path, duration of execution of the project and calculation of the total cost.

2) Solution of the network with the maximum durations of the activities which correspond to their minimum direct cost. Calculation of all time and costs elements of the network.

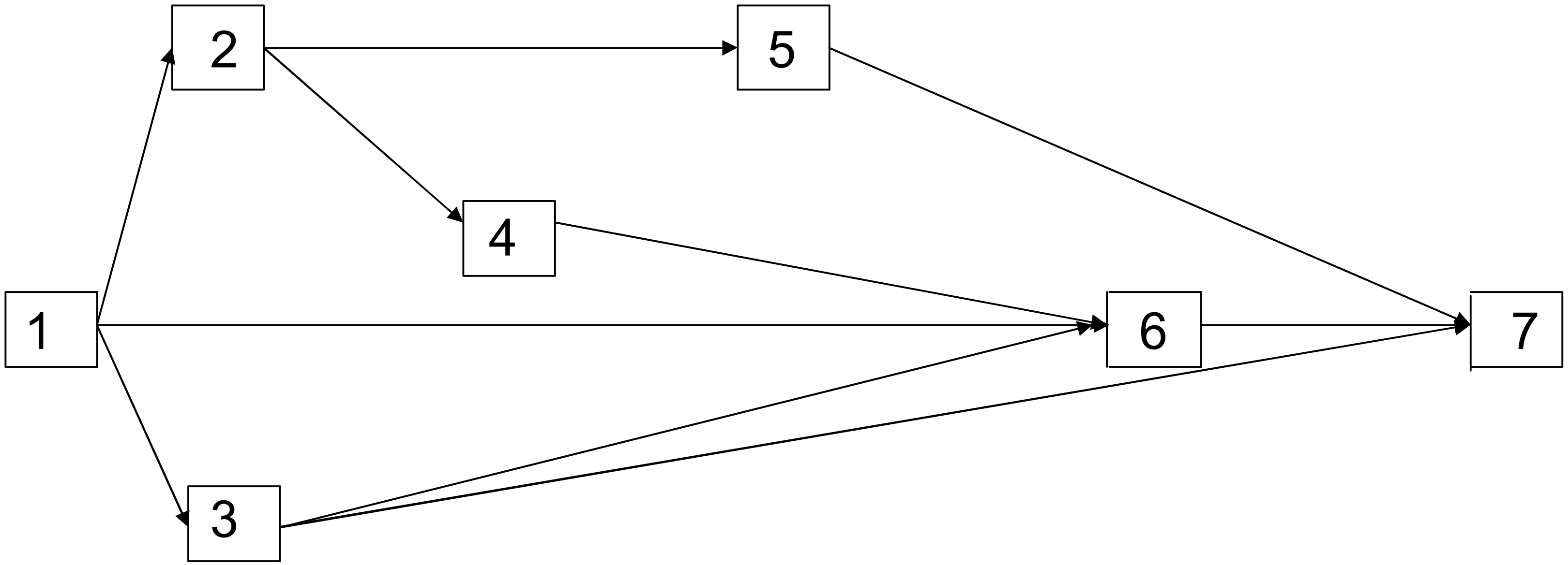
3) Reduction of the total duration of step 2 by one time unit. This can be achieved by reducing by one time unit the duration of the critical activity having the lowest cost coefficient c(i,j). Thus, the direct cost is increased by the minimum possible amount.

4) Solution of the network set up at step 3 and calculation of the time and cost elements.

5) The project duration is reduced successively by one time unit, until it its duration reaches the minimum. In each reduction of project duration is selected to be reduced the activity of the critical path with the minimum c(i,j) (or in case the corresponding network has more than one critical paths the critical activities – one of every critical path - with the minimum sum of cost coefficients).

6) As the optimum duration of the project is selected the corresponding to its minimum total cost. Therefore, the project should be planned so that each activity is carried out with duration equal to its duration corresponding to the network with the minimum total cost.

1st example of CPM method application

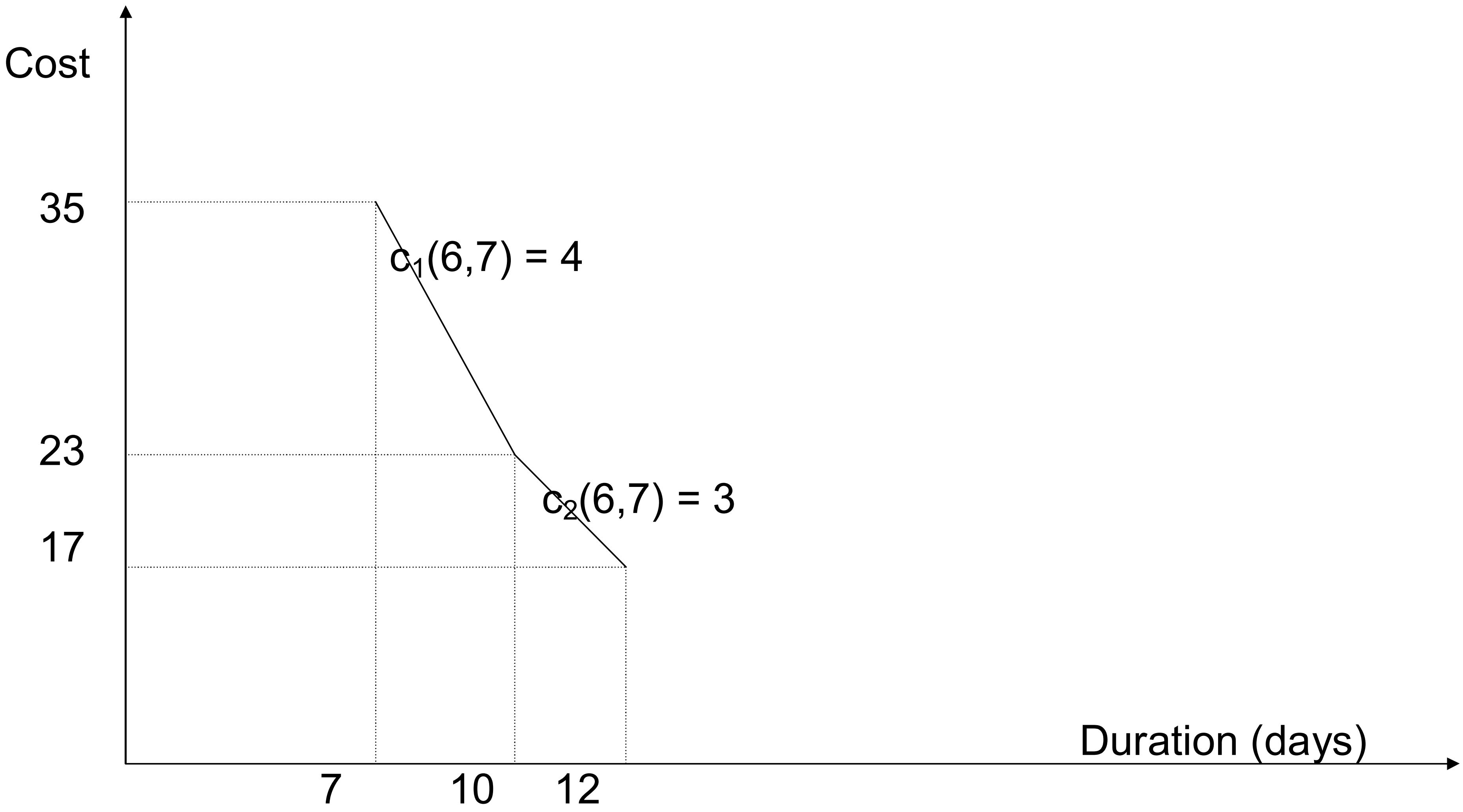


Network of a ship's equipment procurement and installation project

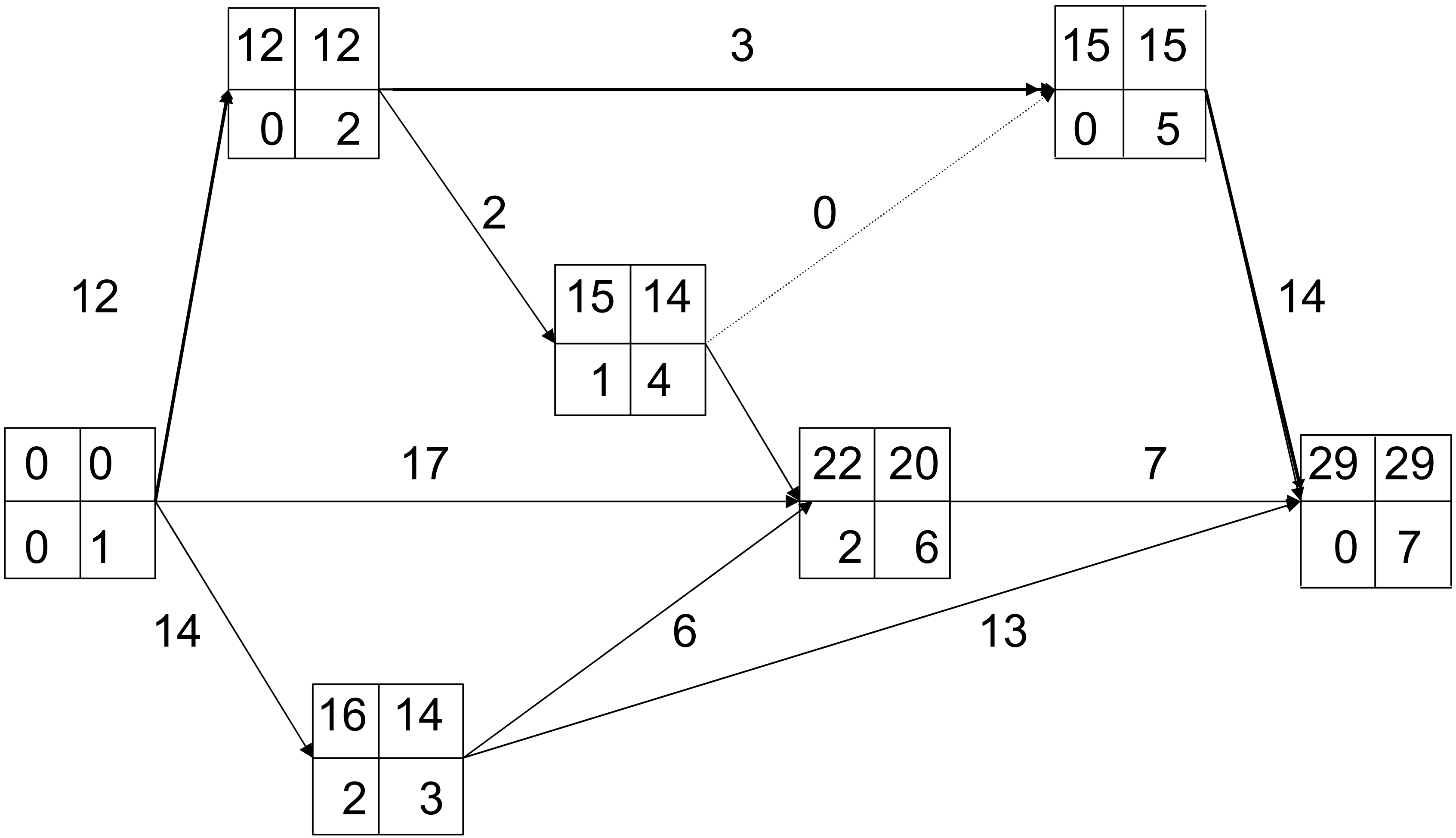
Activities of ship's equipment procurement and installation

Activity (i,j)	Maximum duration $t_{max}(i,j)$	Minimum cost $K_{min}(i,j)$	Minimum duration $t_{min}(i,j)$	Maximum cost $K_{max}(i,j)$
1-2	15	30	12	39
1-6	20	40	17	49
1-3	15	10	14	15
2-5	5	10	3	20
2-4	3	5	2	16
3-6	7	8	6	10
3-7	17	30	13	38
4-6	8	15	6	25
4-5	0	0	0	0
5-7	16	10	14	12
6-7	12	17	10	23
6-7			7	35

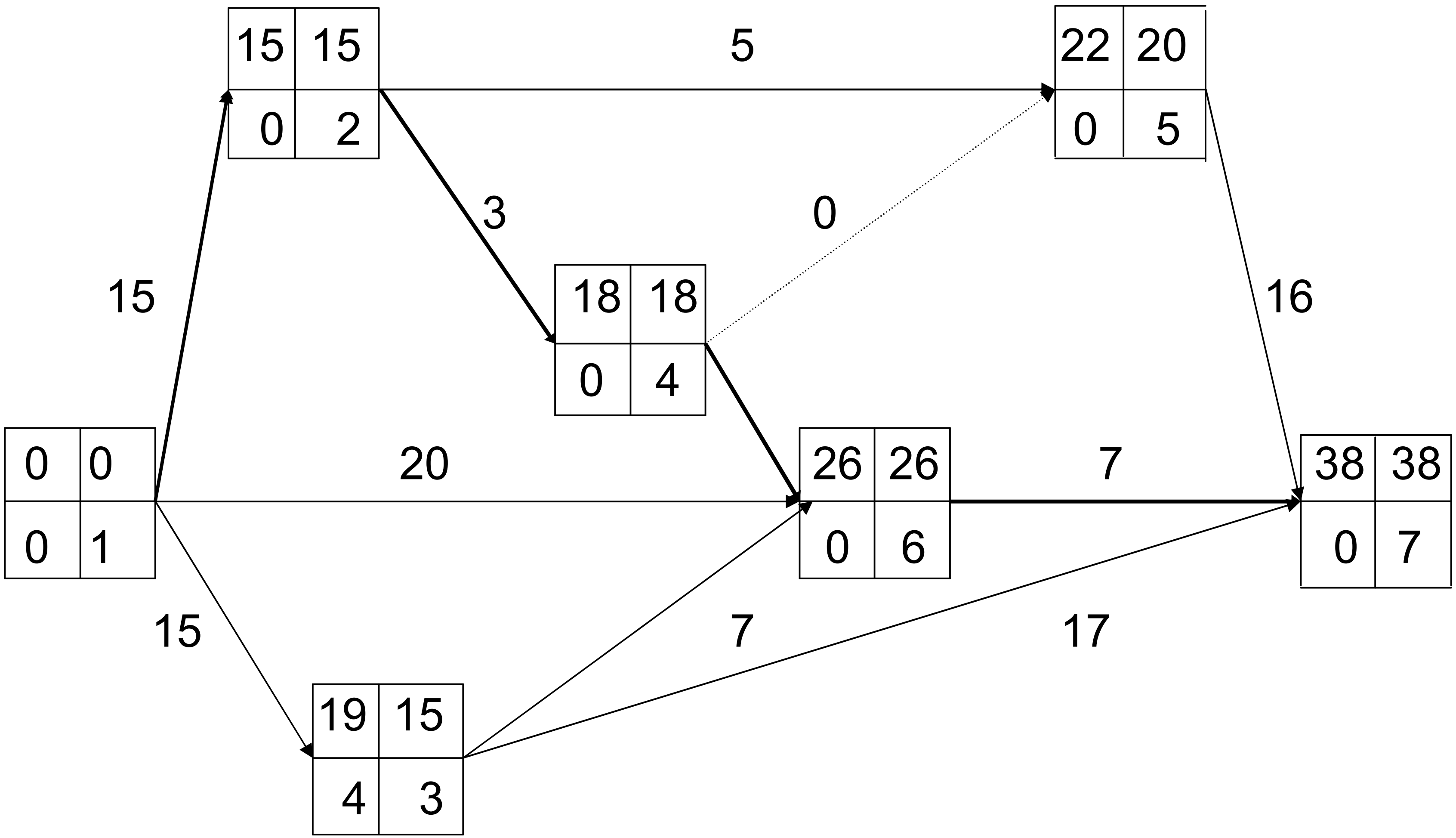
Durations are expressed in days and costs in thousands of euros



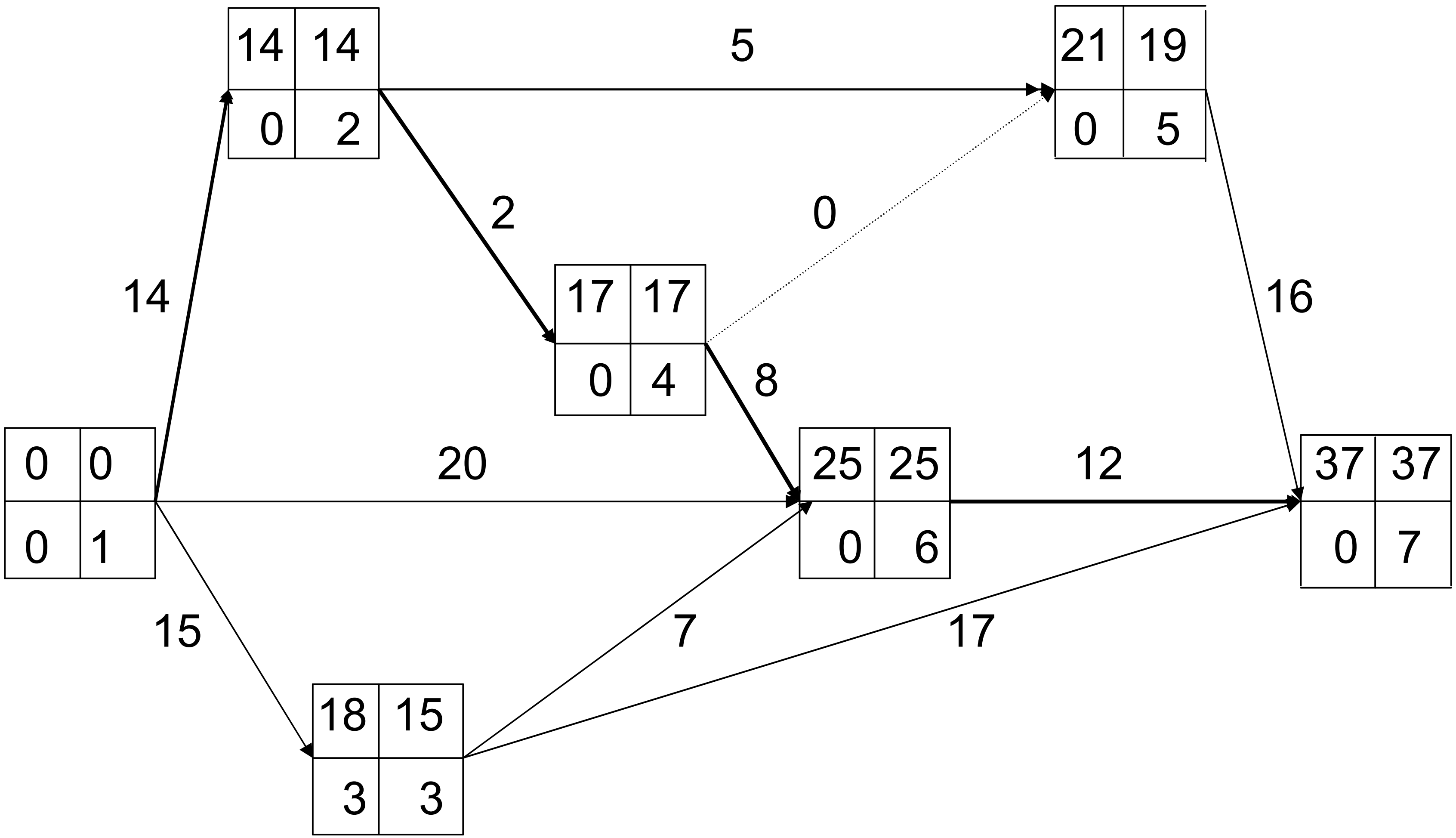
Cost – duration function of activity (6,7)



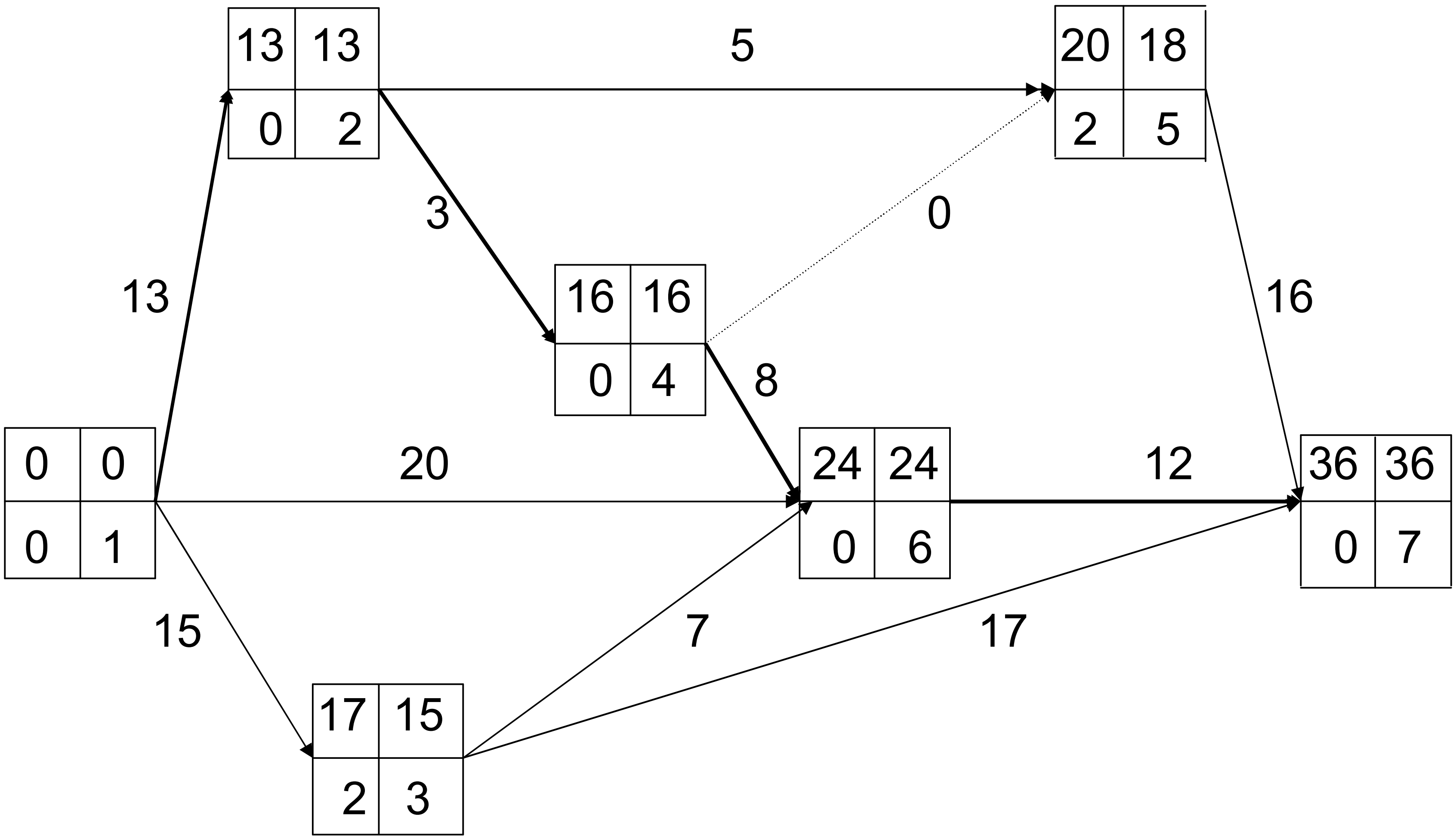
Solution of the network with minimum durations



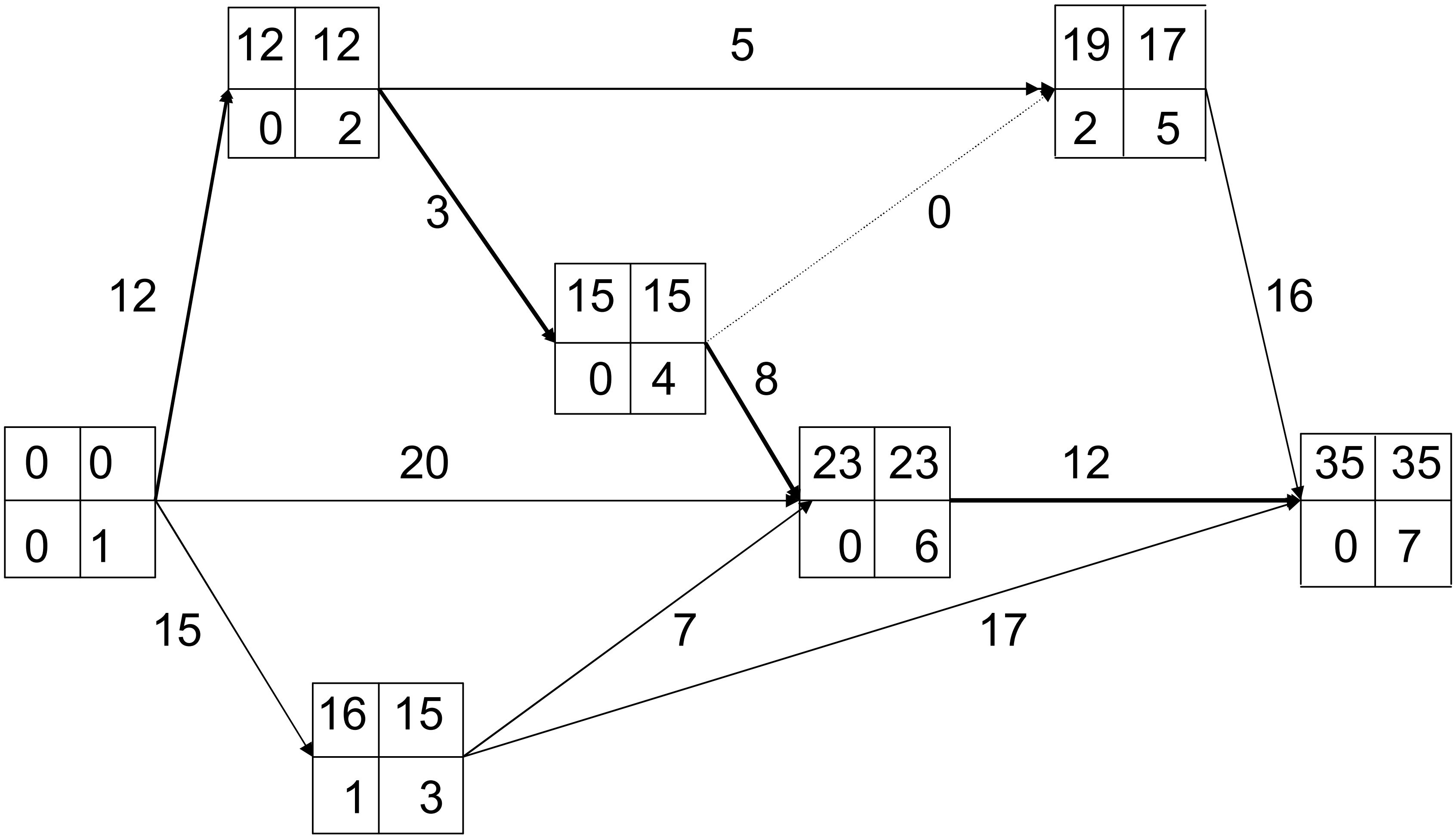
Solution of the network with minimum direct cost (maximum durations)



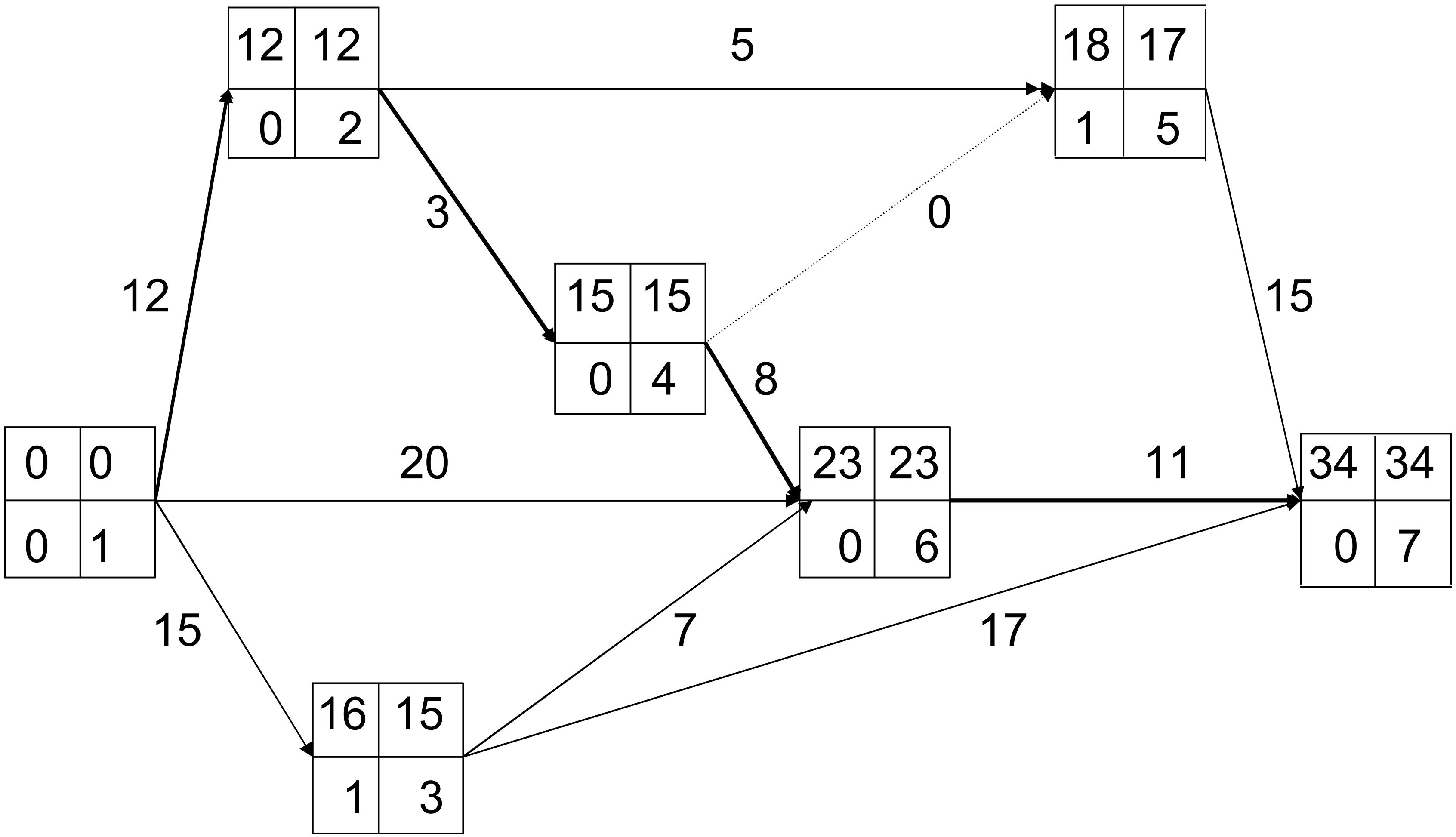
Solution of the network with duration 37 days



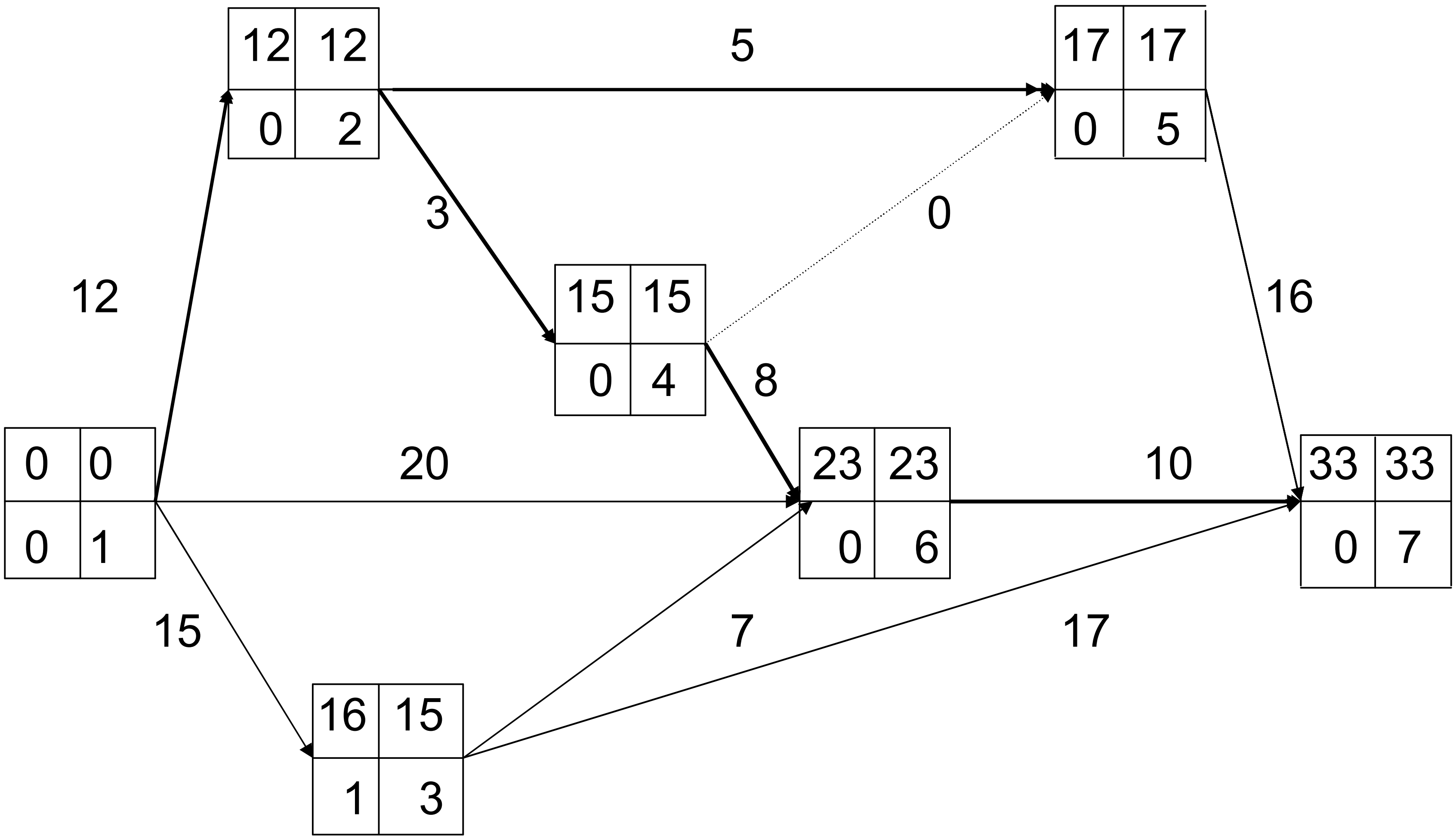
Solution of the network with duration 36 days



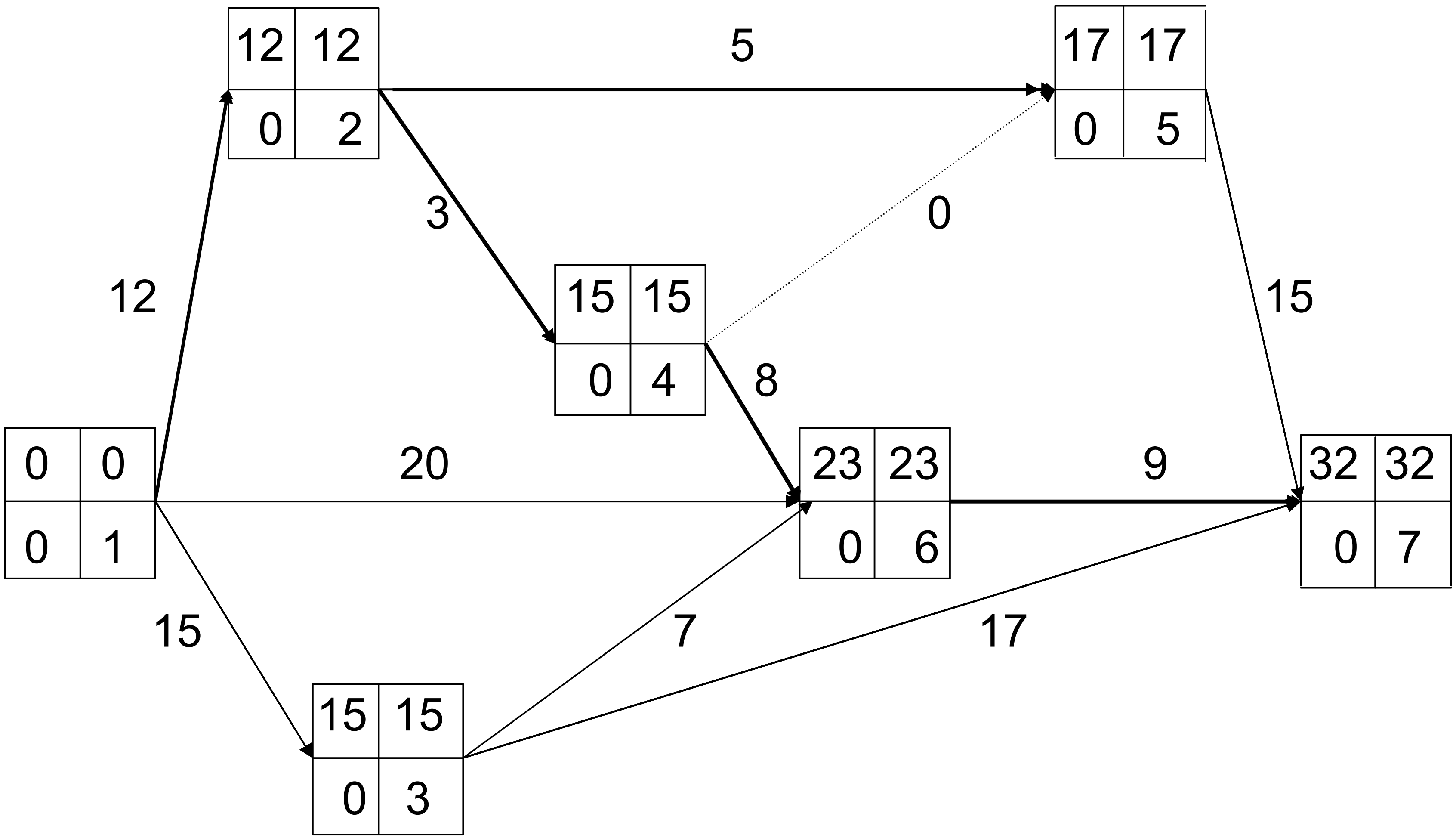
Solution of the network with duration 35 days



Solution of the network with duration 34 days



Solution of the network with duration 33 days



Solution of the network with duration 32 days

Numerical results

Activity (i, j)	$t_{max}(i,j)$	$t_{min}(i,j)$	c(i, j)	Project duration							
				38	37	36	35	34	33	32	31
1-2	15	12	3000	*	3000 *	3000 *	3000 *	*	*	*	
1-3	15	14	5000							*	
1-6	20	17	3000								
2-4	3	2	11000	*	*	*	*	*	*	*	
2-5	5	3	5000							*	
3-6	7	6	2000								
3-7	17	13	2000								2000
4-5	0	0	-								
4-6	8	6	5000	*	*	*	*	*	*	*	
5-7	16	14	1000							1000 *	1000
6-7	12	10	3000	*	*	*	*	3000 *	3000 *		
		7	4000							4000 *	4000
Direct cost $K_d = f_d(T_p)$				175.000	178.000	181.000	184.000	187.000	190.000	195.000	202.000
Indirect cost $K_i = f_i(T_p)$				152.000	148.000	144.000	140.000	136.000	132.000	128.000	124.000
Total cost $K = f(T_p)$				327.000	326.000	325.000	324.000	323.000	322.000	323.000	326.000

↑ Optimal solution

2^{nd} example of CPM technique application

Activity	Normal time (N_i)	Compressed time (C_i)	Normal cost (NC_i)	Maximum compression cost (CC_i)	Compression cost per day (RC_i)
A	15	10	10	15	1
B	10	6	5	9	1
C	5	2	80	140	20
D	18	11	50	85	5
E	3	1	10	30	10
F	8	4	20	28	2
G	10	7	15	30	5
H	12	6	30	54	4
I	7	4	5	14	3

Results of CPM method with normal data (WinQSB)

(normal costs and durations)

	Activity Name	On Critical Path	Activity Time	Earliest Start	Earliest Finish	Latest Start	Latest Finish	Slack (LS-ES)
1	A	Yes	15	0	15	0	15	0
2	B	Yes	10	15	25	15	25	0
3	C	no	5	15	20	20	25	5
4	D	Yes	18	25	43	25	43	0
5	E	no	3	20	23	62	65	42
6	F	no	8	43	51	45	53	2
7	G	Yes	10	43	53	43	53	0
8	H	Yes	12	53	65	53	65	0
9	I	Yes	7	65	72	65	72	0
	Project	Completion	Time	=	72	days		
	Total	Cost of	Project	=	$225	(Cost on	CP =	$115)
	Number of	Critical	Path(s)	=	1			

Results of CPM method with complete compression data

(compressed costs and durations)

	Activity Name	On Critical Path	Activity Time	Earliest Start	Earliest Finish	Latest Start	Latest Finish	Slack (LS-ES)
1	A	Yes	10	0	10	0	10	0
2	B	Yes	6	10	16	10	16	0
3	C	no	2	10	12	14	16	4
4	D	Yes	11	16	27	16	27	0
5	E	no	1	12	13	39	40	27
6	F	no	4	27	31	30	34	3
7	G	Yes	7	27	34	27	34	0
8	H	Yes	6	34	40	34	40	0
9	I	Yes	4	40	44	40	44	0
	Project	Completion	Time	=	44	days		
	Total	Cost of	Project	=	$405	(Cost on	CP =	$207)
	Number of	Critical	Path(s)	=	1			

Compression to 44 days using the compression algorithm

(calculating the real required additional cost)

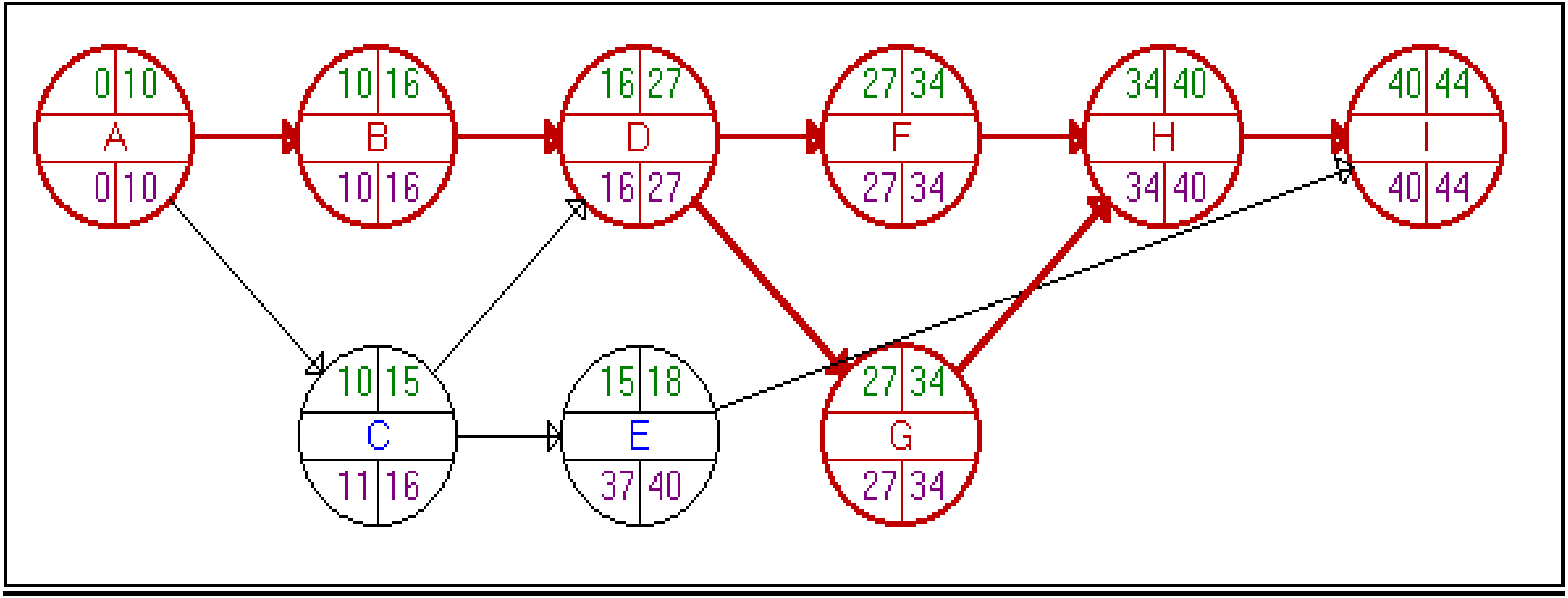
	Activity Name	Critical Path	Normal Time	Crash Time	Suggested Time	Additional Cost	Normal Cost	Suggested Cost
1	A	Yes	15	10	10	$5	$10	$15
2	B	Yes	10	6	6	$4	$5	$9
3	C	no	5	2	5	0	$80	$80
4	D	Yes	18	11	11	$35	$50	$85
5	E	no	3	1	3	0	$10	$10
6	F	Yes	8	4	7	$2	$20	$22
7	G	Yes	10	7	7	$15	$15	$30
8	H	Yes	12	6	6	$24	$30	$54
9	I	Yes	7	4	4	$9	$5	$14
	Overall	Project:			44	$94	$225	$319

Remaining calculations of CPM method for 44 days

	Activity Name	On Critical Path	Activity Time	Earliest Start	Earliest Finish	Latest Start	Latest Finish	Slack (LS-ES)
1	A	Yes	10	0	10	0	10	0
2	B	Yes	6	10	16	10	16	0
3	C	no	5	10	15	11	16	1
4	D	Yes	11	16	27	16	27	0
5	E	no	3	15	18	37	40	22
6	F	Yes	7	27	34	27	34	0
7	G	Yes	7	27	34	27	34	0
8	H	Yes	6	34	40	34	40	0
9	I	Yes	4	40	44	40	44	0
	Project	Completion	Time	=	44	days		
	Total	Cost of	Project	=	$319	(Cost on	CP =	$229)
	Number of	Critical	Path(s)	=	2			

Critical paths for 44 days:

- A → B → D → F → H → I (cost = 199)
- A → B → D → G → H → I (cost = 207)



Step by step compression to 59 days

	Activity Name	Critical Path	Normal Time	Crash Time	Suggested Time	Additional Cost	Normal Cost	Suggested Cost
1	A	Yes	15	10	10	$5	$10	$15
2	B	Yes	10	6	6	$4	$5	$9
3	C	no	5	2	5	0	$80	$80
4	D	Yes	18	11	18	0	$50	$50
5	E	no	3	1	3	0	$10	$10
6	F	no	8	4	8	0	$20	$20
7	G	Yes	10	7	10	0	$15	$15
8	H	Yes	12	6	11	$4	$30	$34
9	I	Yes	7	4	4	$9	$5	$14
	Overall	Project:			59	$22	$225	$247

1st, unit cost = 1

2nd, unit cost = 1

3rd, unit cost = 3

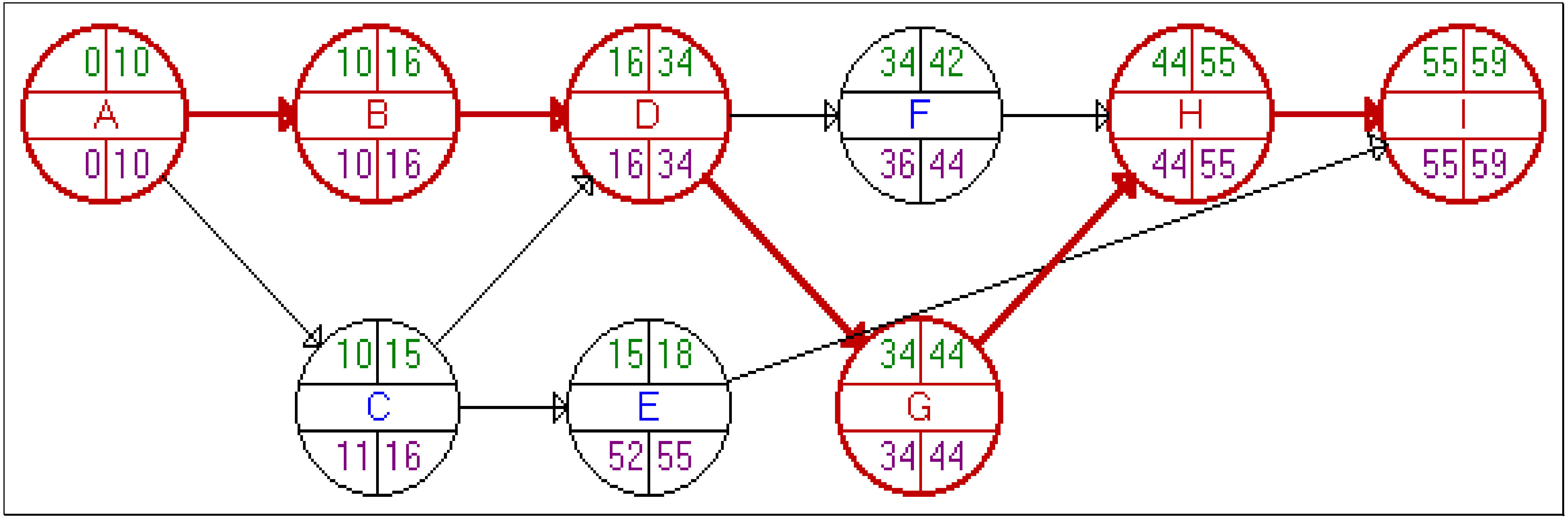
4th ,unit cost = 4

Results of CPM method calculations for 59 days

	Activity Name	On Critical Path	Activity Time	Earliest Start	Earliest Finish	Latest Start	Latest Finish	Slack (LS-ES)
1	A	Yes	10	0	10	0	10	0
2	B	Yes	6	10	16	10	16	0
3	C	no	5	10	15	11	16	1
4	D	Yes	18	16	34	16	34	0
5	E	no	3	15	18	52	55	37
6	F	no	8	34	42	36	44	2
7	G	Yes	10	34	44	34	44	0
8	H	Yes	11	44	55	44	55	0
9	I	Yes	4	55	59	55	59	0
	Project	Completion	Time	=	59	days		
	Total	Cost of	Project	=	$247	(Cost on	CP =	$137)
	Number of	Critical	Path(s)	=	1			

Critical path for 59 days:

A → B → D → G → H → I (cost = 137)



Variation: total budget = 300 cost units

	Activity Name	Critical Path	Normal Time	Crash Time	Suggested Time	Additional Cost	Normal Cost	Suggested Cost
1	A	Yes	15	10	10	$5	$10	$15
2	B	Yes	10	6	6	$4	$5	$9
3	C	no	5	2	5	0	$80	$80
4	D	Yes	18	11	12	$30	$50	$80
5	E	no	3	1	3	0	$10	$10
6	F	no	8	4	8	0	$20	$20
7	G	Yes	10	7	10	0	$15	$15
8	H	Yes	12	6	6	$24	$30	$54
9	I	Yes	7	4	4	$9	$5	$14
	Overall	Project:			48	$72	$225	$297

5th, unit cost = 5

4th, unit cost = 4, running out

Continuing compression in integer time units

	Activity Name	Critical Path	Normal Time	Crash Time	Suggested Time	Additional Cost	Normal Cost	Suggested Cost
1	A	Yes	15	10	10	$5	$10	$15
2	B	Yes	10	6	6	$4	$5	$9
3	C	no	5	2	5	0	$80	$80
4	D	Yes	18	11	11	$35	$50	$85
5	E	no	3	1	3	0	$10	$10
6	F	no	8	4	8	0	$20	$20
7	G	Yes	10	7	10	0	$15	$15
8	H	Yes	12	6	6	$24	$30	$54
9	I	Yes	7	4	4	$9	$5	$14
	Overall	Project:			47	$77	$225	$302

Compression in 47 days has total cost 302 (i.e. 297 + 5).

So, the budget cannot cover the cost of one extra day.

Compression exactly on the bound of 300 monetary units

	Activity Name	Critical Path	Normal Time	Crash Time	Suggested Time	Additional Cost	Normal Cost	Suggested Cost
1	A	Yes	15	10	10	$5	$10	$15
2	B	Yes	10	6	6	$4	$5	$9
3	C	no	5	2	5	0	$80	$80
4	D	Yes	18	11	11.4	33.00	$50	83.00
5	E	no	3	1	3	0	$10	$10
6	F	no	8	4	8	0	$20	$20
7	G	Yes	10	7	10	0	$15	$15
8	H	Yes	12	6	6	$24	$30	$54
9	I	Yes	7	4	4	$9	$5	$14
	Overall	Project:			47.40	75.00	$225	300.00

We gain 0.6 more days, with cost 0.6 × 5 = 3 (+297 = 300)

Totally, compression of D per 6.6 days with cost 33 monetary units

Typical questions that can be investigated with CPM

1. If the project has to be completed until a specified time limit which is less than the duration of the project that has been calculated with the normal activity times:

   Which activities must be compressed, for how many time units and how much will the cost be to complete the project within the specified time limit?

2. If we have a specific budget with which we try to reduce the duration of the project:

   Up to how many time units can we reduce the duration of the project having this budget?

Advantages of PERT and CPM methods

- Extremely useful when scheduling and controlling large projects.
- Straightforward concepts and not mathematically complex.
- Graphical representations (networks) of projects aiming to the perception of relationships among project activities.
- Critical path & slack times analyses aim to the identification of activities, which need to be closely watched.
- Project documentation and graphics point who is responsible for each activity.
- Applicable to a wide variety of projects.

Limitations of PERT and CPM methods

- Assume clearly defined, independent and stable activities.

- Specified precedence relationships among the activities.

- In PERT technique activity times follow Beta distribution with subjective time estimates.

- Overemphasis on the critical path, ignoring other “sub-critical” paths which may last slightly less than the critical and it is likely to surpass it in duration.

Synopsis

- A project is a unique, one-time event of some duration consuming resources and it has been designed to achieve an objective in a given time period.

- Each project goes through a five-phase life cycle: concept, feasibility study, planning, execution, and termination.

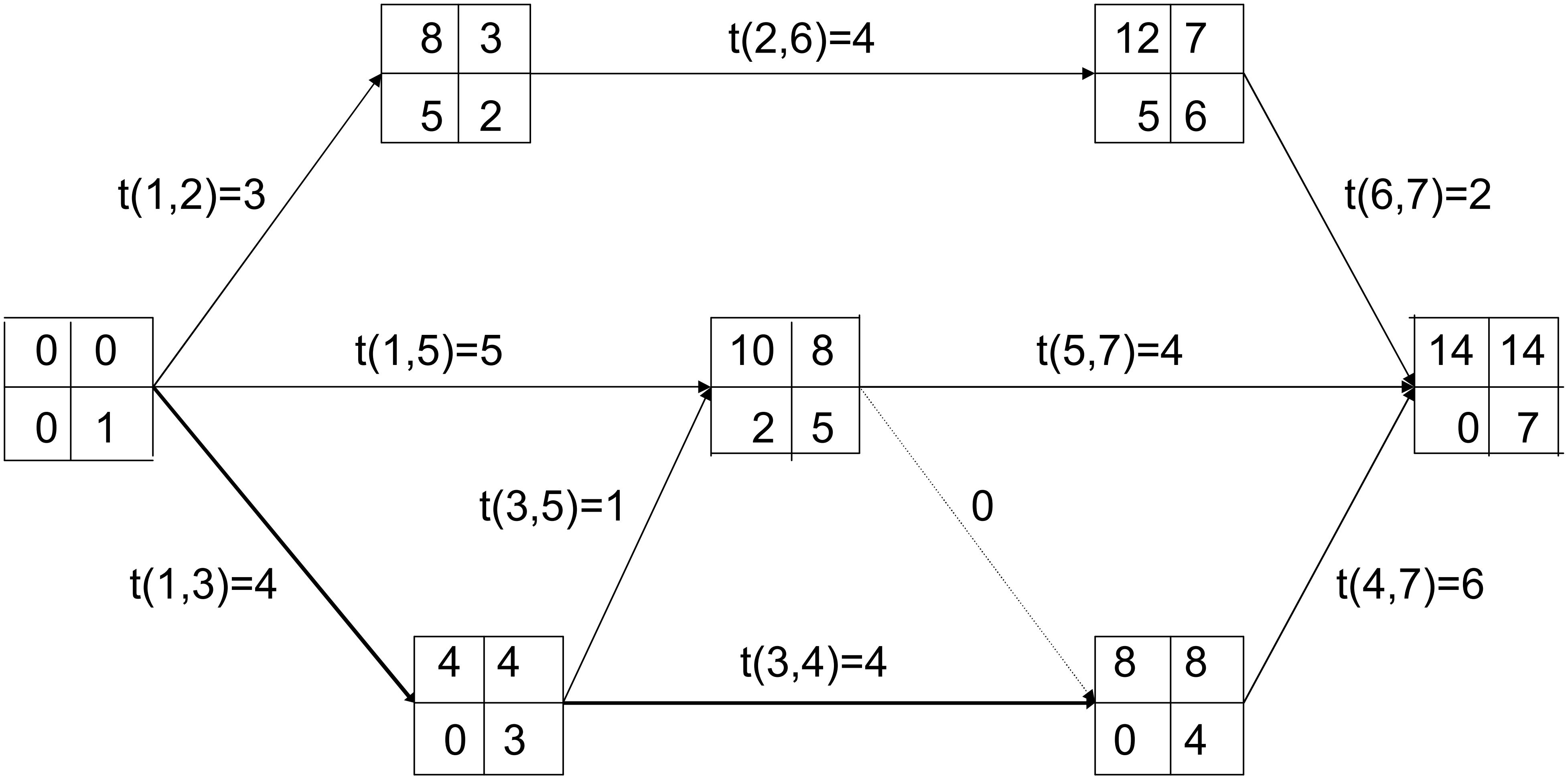
- The two main network planning techniques are PERT and CPM:
  - PERT uses probabilistic time estimates.
  - CPM uses deterministic time estimates.

- PERT and CPM determine the critical path of the project and the estimated completion time. On large projects, software programs are available to identify the critical path, minimize the cost and plan the use of the available resources.

- PERT uses probabilistic time estimates to determine the probability that a project will be completed in a specific time.

- To reduce the length of the project (crashing), the critical path of the project and the cost of reducing individual activity times have to be determined. Crashing the duration of activities which do not belong to the critical path, typically does not reduce the project completion time.

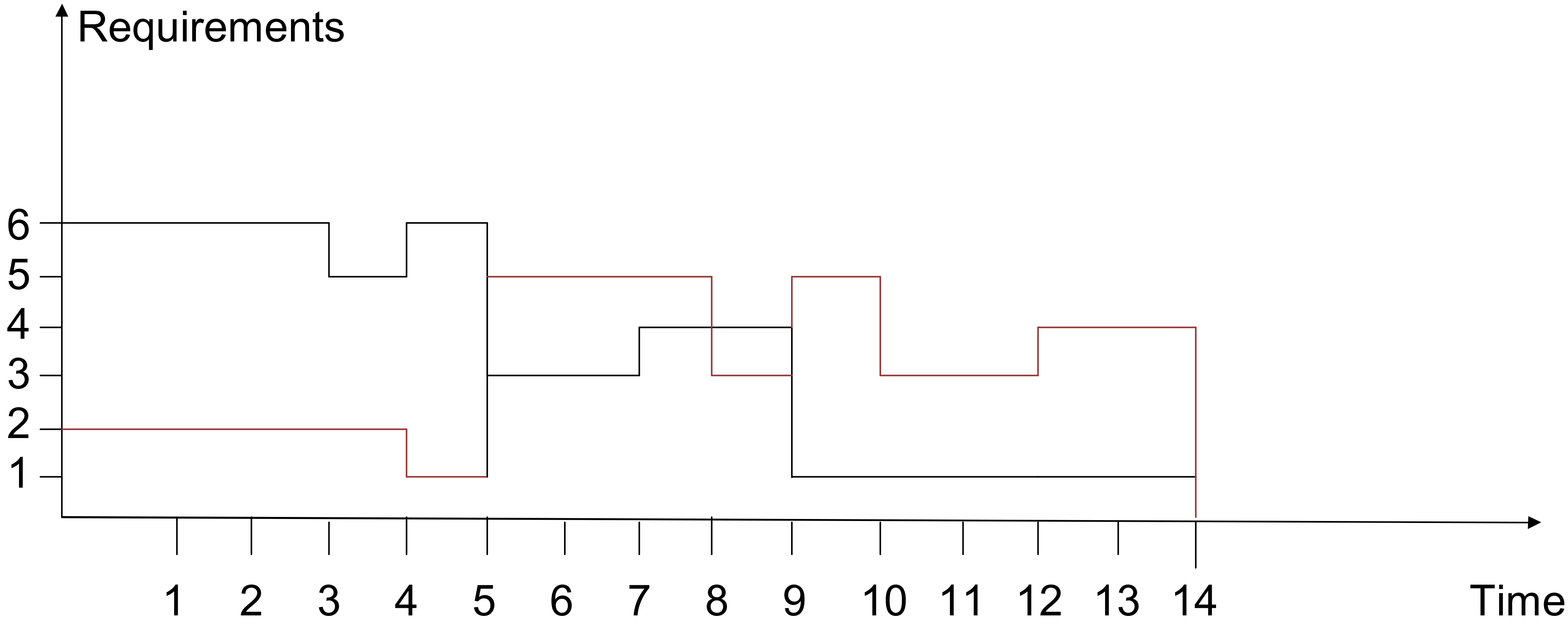
Resource scheduling



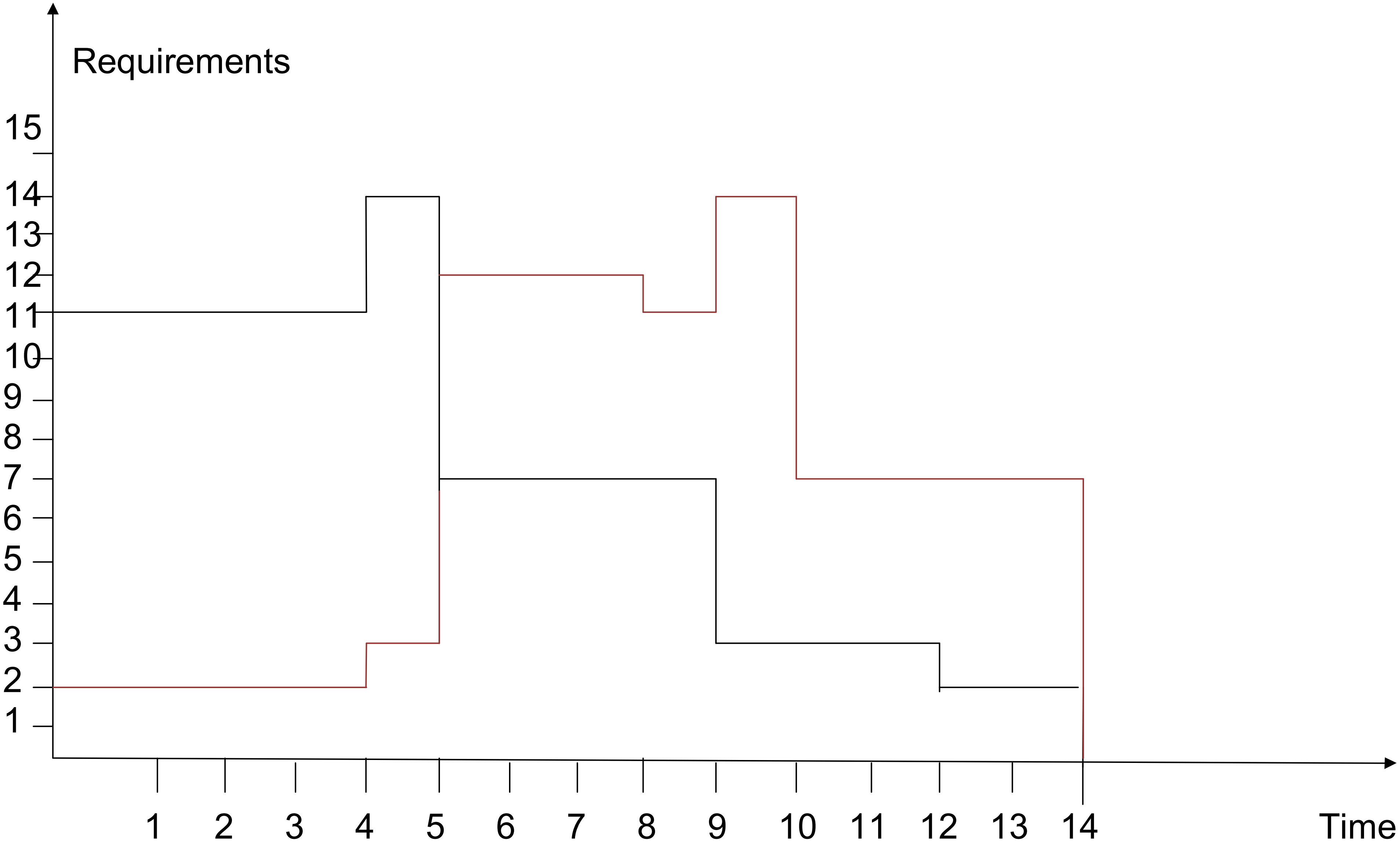
Example of Gantt diagram network

Gantt diagram

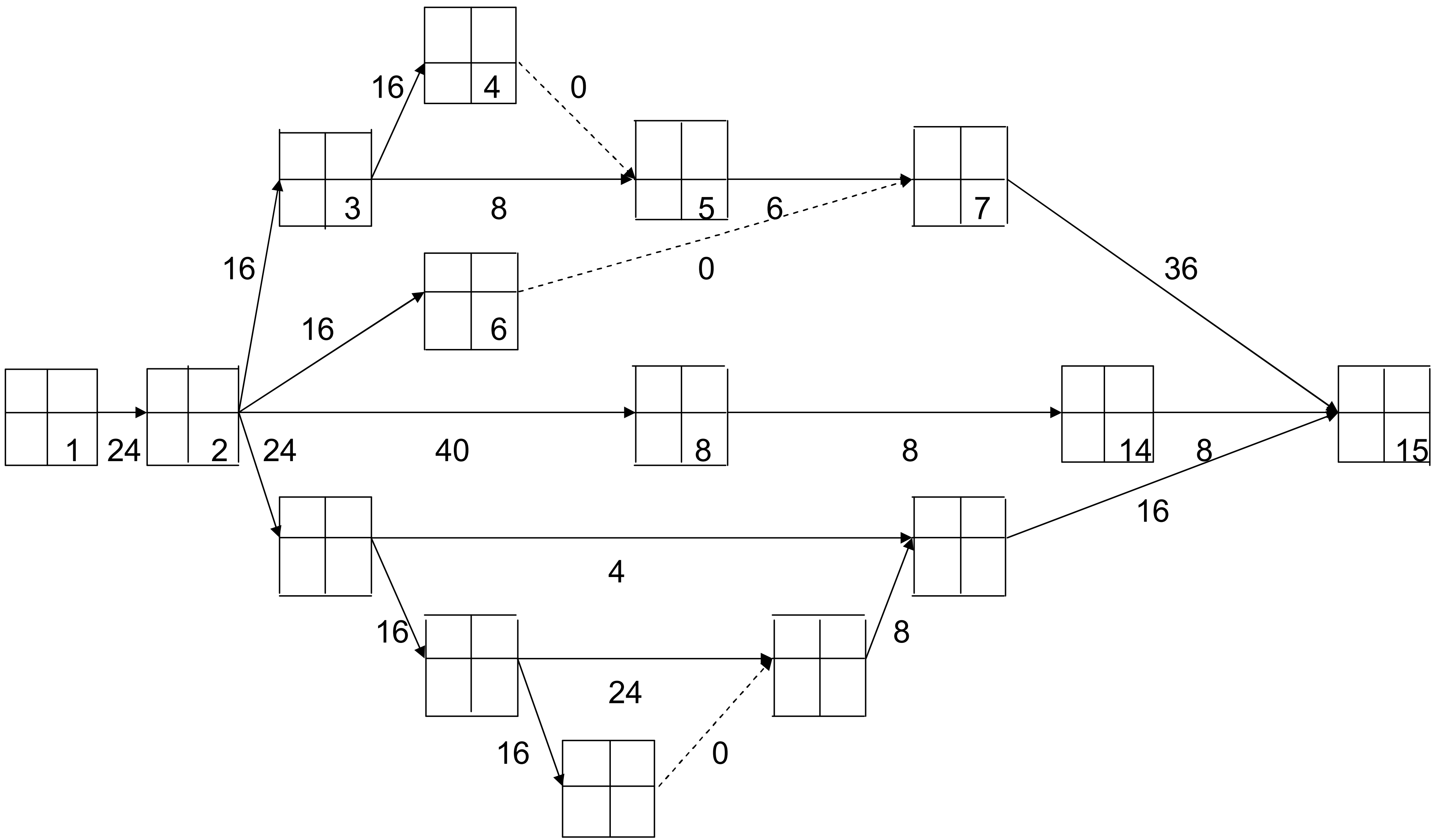
Event		Duration		Time periods													
i	j	t(i,j)	R1-R2	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	3	4	2-2	2-2	2-2	2-2	2-2										
3	4	4	1-3					1-3	1-3	1-3	1-3						
4	7	6	0-2									0-2	0-2	0-2	0-2	0-2	0-2
1	2	3	3-4	[3-4]	[3-4]	[3-4]			(3-4)	(3-4)	(3-4)						
1	5	5	1-5	[1-5]	[1-5]	[1-5]	[1-5]	[1-5]	(1-5)	(1-5)	(1-5)	(1-5)	(1-5)				
2	6	4	2-4				[2-4]	[2-4]	[2-4]	[2-4]		(2-4)	(2-4)	(2-4)	(2-4)		
3	5	1	2-3					[2-3]					(2-3)				
6	7	2	3-4								[3-4]	[3-4]				(3-4)	(3-4)
5	7	4	1-1									[1-1]	[1-1]	1-1	1-1	(1-1)	(1-1)
Resources R1-R2 for earlier start times				6-11	6-11	6-11	5-11	6-15	3-7	3-7	4-7	+ 4-7	1-3	1-3	1-3	0-2	0-2
Resources R1-R2 for latest start times				2-2	2-2	2-2	2-2	1-3	5-12	5-12	5-12	3-11	5-14	3-7	3-7	4-7	4-7



Requirement diagram of first type of resources (R1)



Requirement diagram of second type of resources (R2)



Refinery unit maintenance project

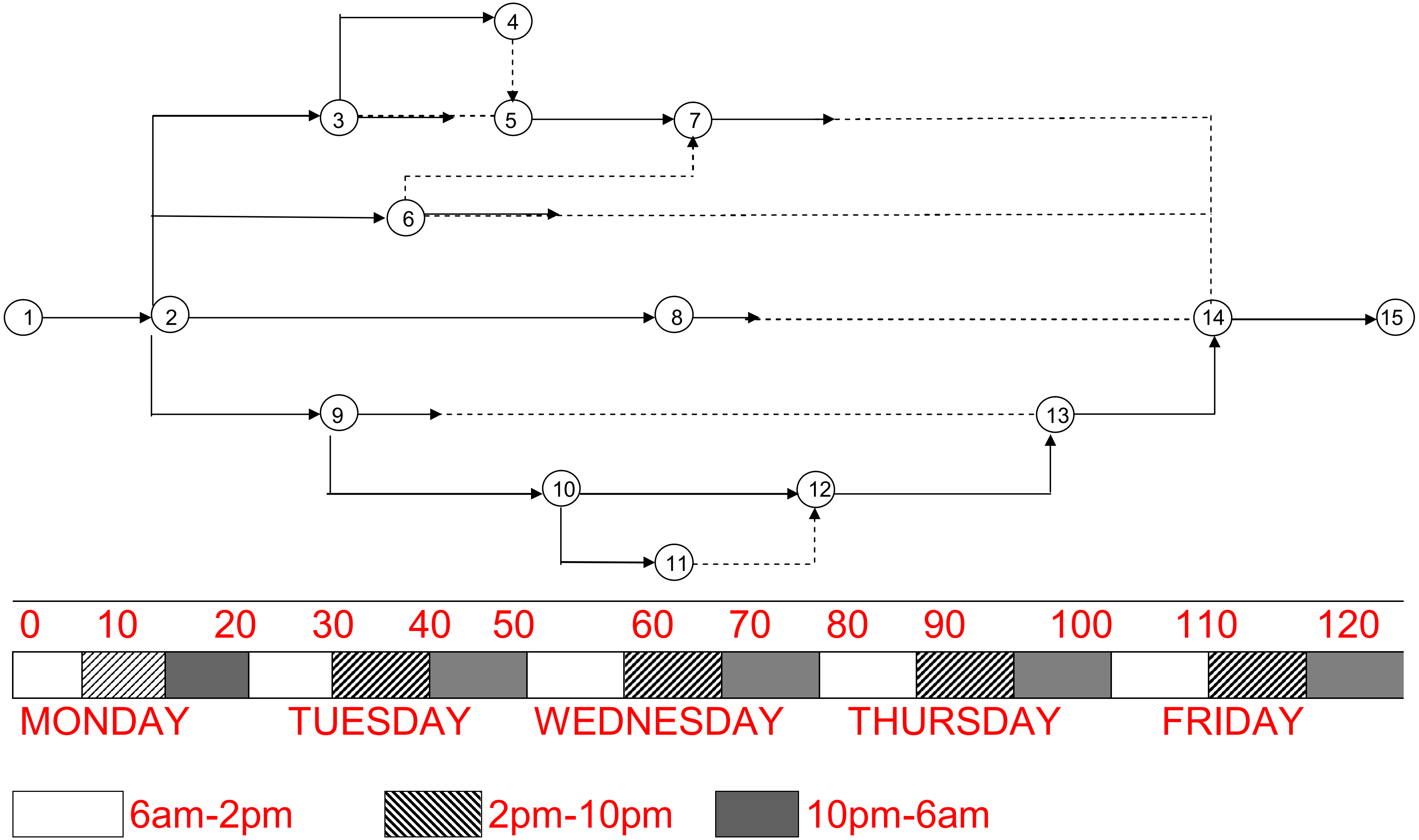
Refinery unit maintenance project
Activities' times and floats

Project			Event times				Critical Path	Time floats	
Events i	j	Duration (hours)	ES	EF	LS	LF		Total	Free
1	2	24	0	24	0	24	√	0	0
2	3	16	24	40	38	54		14	0
2	6	16	24	40	60	76		36	0
2	8	40	24	64	64	104		40	0
2	9	24	24	48	24	48	√	0	0
3	4	16	40	56	54	70		14	0
3	5	8	40	48	62	70		22	8
4	5	0	56	56	70	70		14	0
5	7	6	56	62	70	76		14	0
6	7	0	40	40	76	76		36	22
6	14	12	40	52	100	112		60	60
7	14	36	62	98	76	112		14	14
8	14	8	64	72	104	112		40	40
9	10	16	48	64	48	64	√	0	0
9	13	4	48	52	92	96		44	44
10	11	16	64	80	72	88		8	0
10	12	24	64	88	64	88	√	0	0
11	12	0	80	80	88	88		8	8
12	13	8	88	96	88	96	√	0	0
13	14	16	96	112	96	112	√	0	0
14	15	8	112	120	112	120	√	0	0

Resource allocation

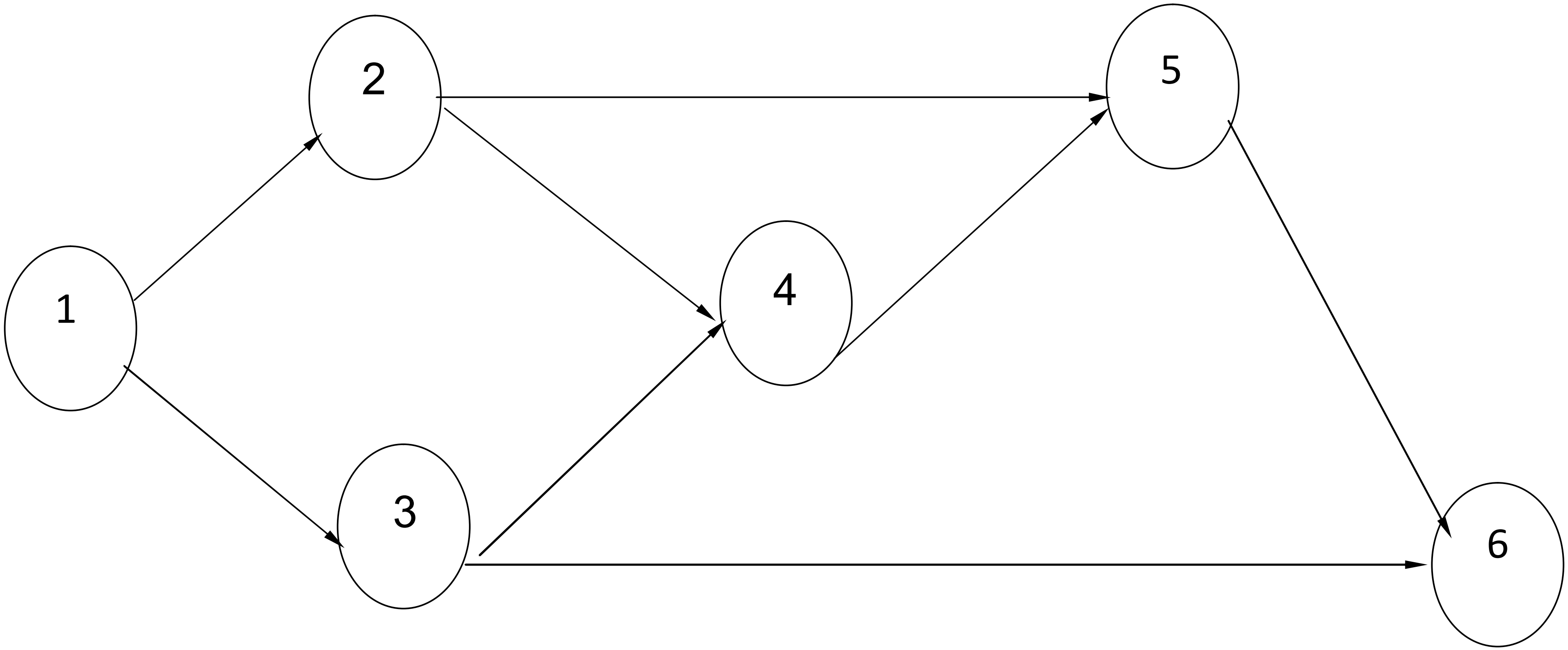
Project			Total floats									
i	j	t	TF	0	24	40	48	56	64	New TF	72	New TF
1	2	24	0	0	∀							
2	3	16	14		14	∀						
2	6	16	36		36	20	12	<u>4</u>			∀	
2	8	40	40		40	24	16	8	0	0	-6	<u>0</u>
2	9	24	0		<u>0</u>		N					
3	4	16	14			14		∀				
3	5	8	22			22	14	6	-2	2	∀	
4	5	0	14					∀				
5	7	6	14								0	6
6	7	0	36								∀	
6	14	12	60								30	36
7	14	36	14									
8	14	8	40									
9	10	16	0				0	∀				
9	13	4	44				44	36	28	30	22	28
10	11	16	8						8	10	2	8
10	12	24	0						0	2	-6	<u>0</u>
11	12	0	8									
12	13	8	0									
13	14	16	0									
14	15	8	0									

Total duration 120		+2=122		+6=128		
Foreman Y						
Completion of the project	(1,2)	(2,3)(3,4)	(3,4)	(2,6)	(2,6)	(2,8)
	24	40 56	56	72	72	112
Foreman Z						
Completion of the project		(2,9)(2,9)	(9,10)	(9,10)	(3,5)	(10,12)
		48 48	64	64	72	96
Calendar	MON.	TUE.	WED.	WED.	WED.	THU.
scale	6a.m.	TUE.	6 a.m.	WED.	10p.m	THU.
		6 a.m.		2 p.m.		6 p.m.
		10 p.m.		10 p.m.		6 p.m.



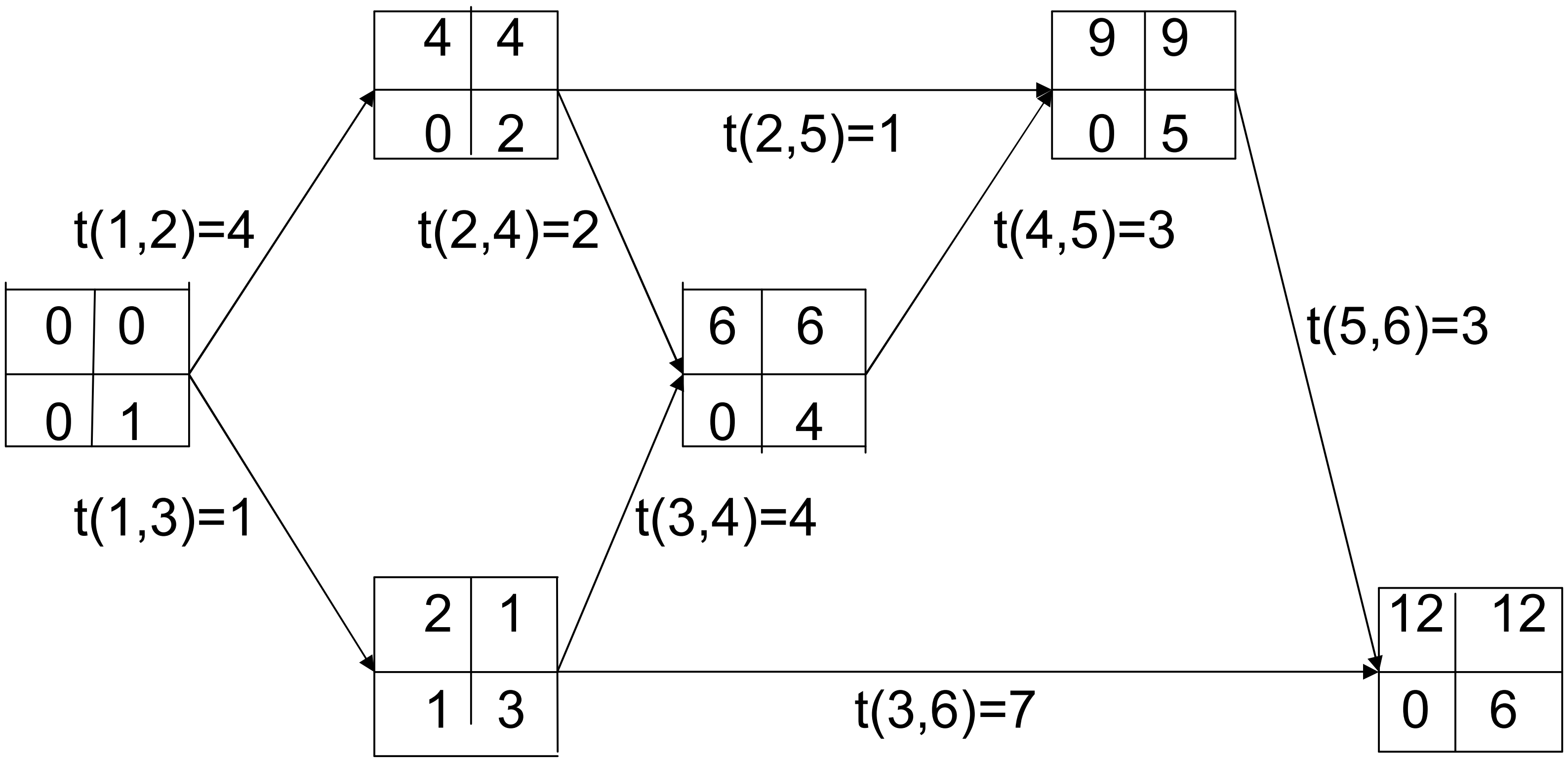
Timetable of the refinery maintenance project

Network of resource smoothing method example



Durations and resources of project's activities

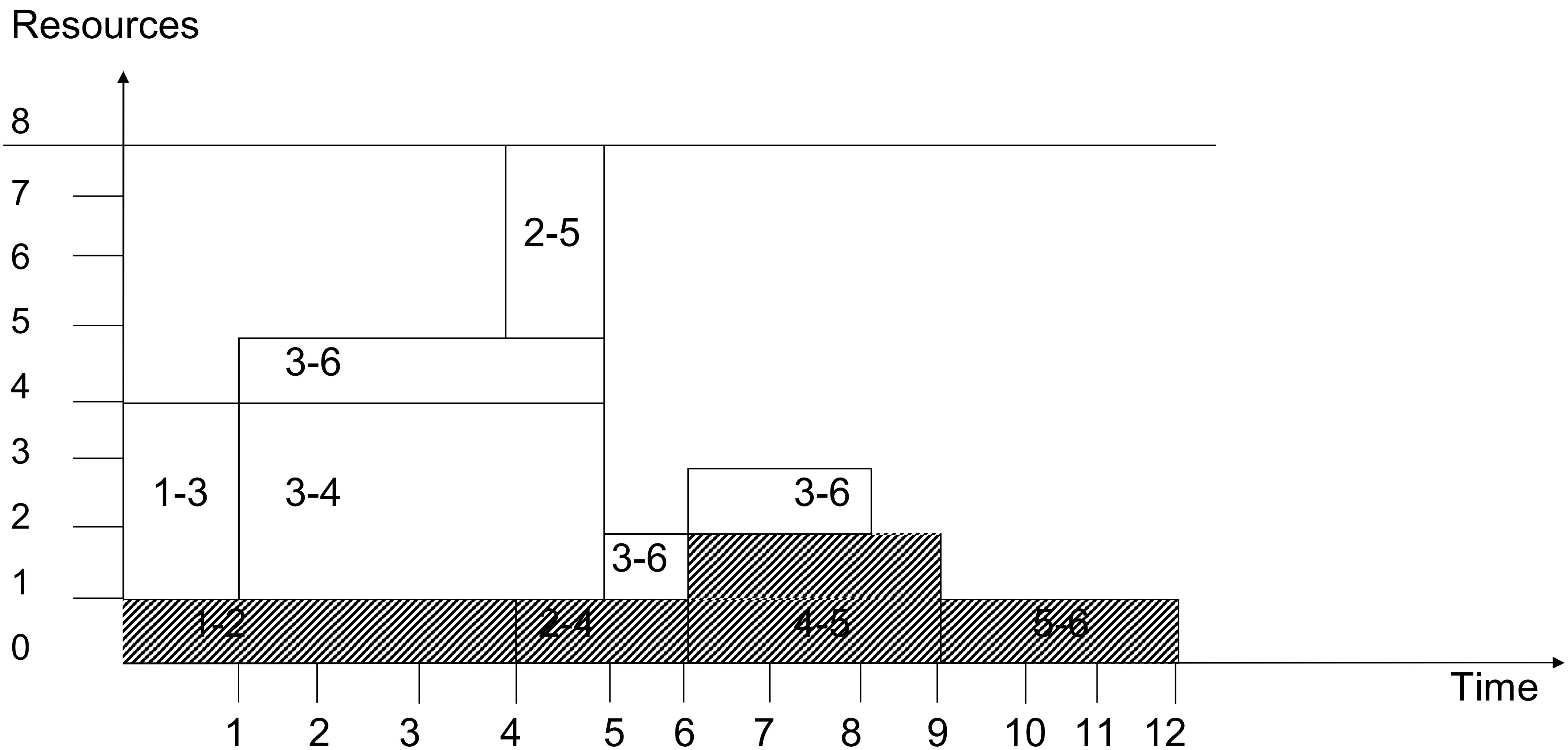
Activities (i, j)	Durations t(i,j)	Human resources r (i,j)
1-2	4	1
1-3	1	3
3-4	4	3
2-4	2	1
4-5	3	2
2-5	1	3
5-6	3	1
3-6	7	1



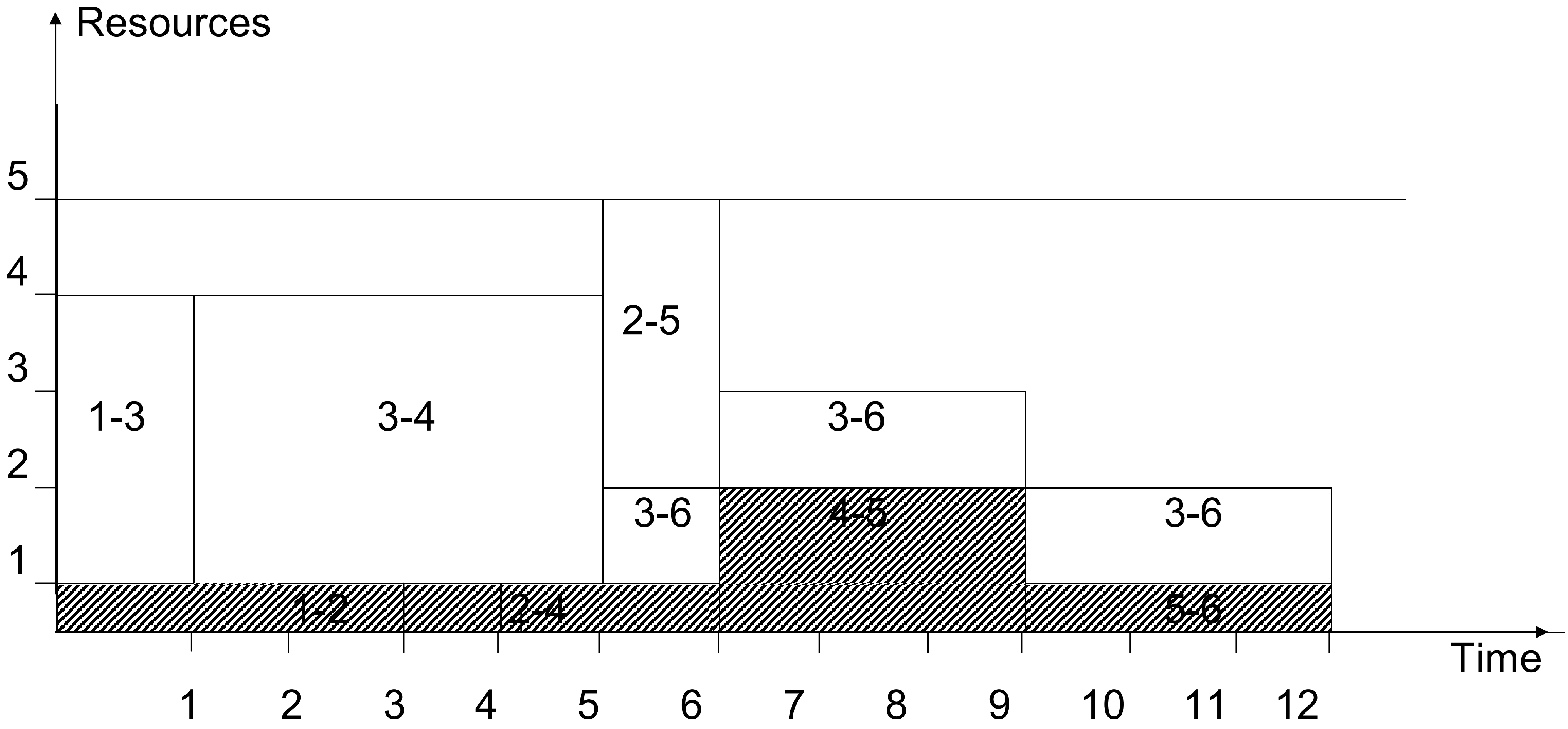
Solution of the network

Activities’ times and floats

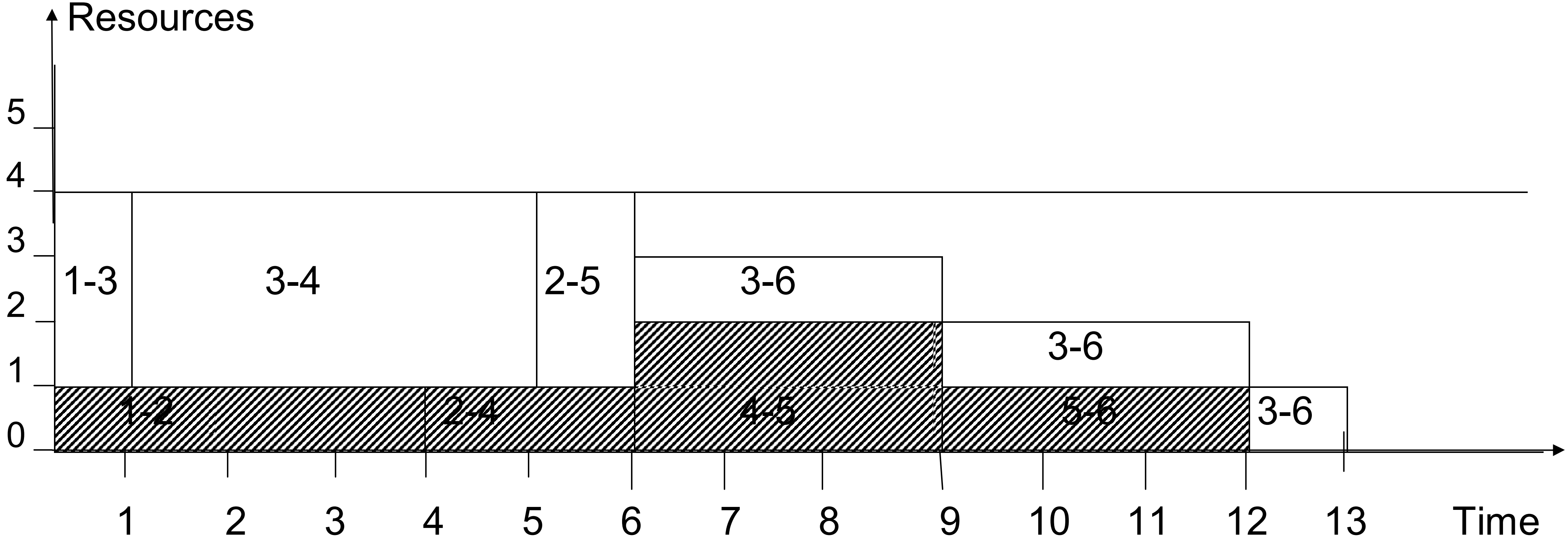
Activity (i,j)	Duration t(i,j)	Resources r(i,j)	ES	EF	LS	LF	Total float
1-2	4	1	0	4	0	4	0 *
1-3	1	3	0	1	1	2	1
2-4	2	1	4	6	4	6	0 *
2-5	1	3	4	5	8	9	4
3-4	4	3	1	5	2	6	1
3-6	7	1	1	8	5	12	4
4-5	3	2	6	9	6	9	0 *
5-6	3	1	9	12	9	12	0 *



Resource aggregation



Resource smoothing



Resource leveling